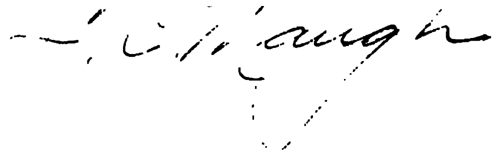

J. C. Haugh

ENGINEERING
FOR
MASONRY DAMS

Kensico Dam of the New York City Water Supply System. *Frontispiece.*

ENGINEERING

FOR

MASONRY DAMS

BY

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PREFACE

In reviewing this book the reader will probably be impressed with the fact that many of the fundamental assumptions of design are based on very obscure data. This is particularly true regarding uplift pressure, ice pressure, and the distribution of stresses in high dams.

It is believed, however, that such assumptions are all on the side of safety; for each of the recorded failures of masonry dams may be attributed to a violation of one or more of the standard rules on which the theory is based.

It is considered that the methods of design described, and the assumptions recommended, represent present conservative practice, and correspond to a proper degree of safety for the average enterprise, and where considerable damage to property and loss of human life would result if failure occurred.

Some of the designing assumptions, and particularly those for the usual unit working stresses, may seem ultra-conservative, as compared with those allowed for other masonry structures. It must be remembered, however, that the theory on which the design of masonry dams is based, is not exact, and, moreover, has not been verified by satisfactory experiments. The failures which have occurred have furnished valuable information regarding some of the limiting conditions, but not all. For instance, no failure due to crushing of concrete masonry has been recorded, and it is not known just what margin of safety the usual working stresses afford. Designs of extremely radical tendencies are being made continually, particularly for dams in unsettled regions, and where there are no government restrictions. There is a possibility, therefore, that future failures will furnish much needed information in this respect.

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Much information was obtained from several branches of the Federal Government, many of the State and Municipal engineering departments, and from engineering literature, as noted in the text.

Acknowledgment is also made to Mr. T. J. McMinn and Mrs. W. P. Creager for invaluable assistance in the editorial work of publication.

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ENGINEERING FOR MASONRY DAMS

CHAPTER I

INVESTIGATIONS AND SURVEYS

1. The Choice of Location. A dam is usually a unit in a more or less extensive project involving the construction of a number of structures of various types. The general location, therefore, is fixed by factors varying with the purpose of the project, and is affected by considerations but remotely allied to the text of this book. Bearing in mind the fact that the general location to be adopted is that which, at reasonable cost, will be best suited to the purpose for which the dam is intended, we have only to consider here those factors which affect the cost and safety of the dam, and the choice of its exact position.

The general location having been chosen, the exact position will be fixed after careful consideration of each of the following factors:

a. The character of the foundation;
b. The configuration of the earth and rock surfaces at the site and its effect on the length of the dam, quantity of material to be excavated, and other factors;
c. Availability and character of materials for construction;
d. The value of the necessary lands and water rights;
e. Requirements as to coffers, pumping, conduits, and other provisions necessary for unwatering the site;
f. Transportation facilities and the accessibility of the site;
g. Availability of suitable sites for construction equipment and camps;
h. The safety of the structure.

The foundation * is one of the most important factors in the final location. It should be practically impervious, or capable

* See Chapter X.

of being made so, and have sufficient strength to sustain the weight of the dam and prevent sliding.

The valley should have sufficient width at the crest of the proposed dam for the requisite spillway,* and in some instances for other structures, such as the power-house in a water-power development. A greater width than that necessary for the spillway and other structures is, of course, undesirable, because of the greater cost of a long dam. The configuration of the earth and solid rock at the site affects greatly the cost of the dam, because of its influence on many items of cost, such as excavation, coffers, and masonry.

The location of sand, and stone or gravel for the masonry, the cost of quarrying, the facilities for transportation to the concrete mixers or stone yard, the quality of such material and its effect on the strength, durability, and appearance of the masonry; the hardness of the stone and its effect on the cost of crushing for concrete or shaping for ashlar facing, all influence the cost of the work and therefore the choice of location.

An important item in the cost of the project will be the value of the lands to be provided for the dam, for the reservoir area, for construction equipment and camps, and for the right of way for a construction highway or railroad to the site. The increase in the elevation of the water surface by the dam may necessitate the relocation of existing railroads or public highways. In some instances the abandonment of towns and villages has been necessary.

The removal of water from the site of the dam, in order to facilitate construction, commonly called "unwatering" the site, often involves a large percentage of the total cost of construction. If the depth of water at the site is considerable, and if it is probable that floods will cause great increases in the stage of the river during the period of construction, the cost of the coffer-dams may be excessive. Many otherwise attractive sites have had to be abandoned on account of the probable great cost of unwatering caused by the great depth and velocity of the water and the difficulties to be overcome in constructing and sealing the coffer-dams.

If the construction is to be accomplished at a reasonable cost, it is important that the site be accessible for the transportation

* See Art. 43.

of plant, materials, and supplies. It is often necessary to construct a highway or a railroad from the nearest shipping point to the site. The cost of constructing, maintaining, and operating the highway or railroad is a controlling factor in the choice of location.

A convenient site of adequate size for the construction equipment and camps is essential for economical construction. The area required for this purpose varies considerably, depending on the size of the structure, and the desired rate of progress in construction. A typical site for the construction plant is a broad, flat stretch of land adjoining and just below the dam site, and above maximum high water. If the materials for the masonry are to be delivered to the dam by cars passing over the coffer-dams, the equipment should be nearly at the same elevation in order to permit of moderate grades. If, however, delivery is to be made by cableway stretched across the valley, or by chutes, or compressed air, most of the plant will be more economical at a higher elevation.

In cold climates, floating ice sometimes enters the reservoir faster than the slack water can carry it to the dam. If the quantity of ice is sufficiently great, and other conditions are favorable, an ice jam will form, and may be greater than has ever been experienced under the natural conditions of the river. This may cause back-water and damage to property, perhaps for miles above. The probability of such ice jams should be given consideration in fixing the site of the dam.

In general, a location below, rather than above, a community of considerable size is to be preferred, principally because of the necessity of providing a greater margin of safety where a failure would result in great destruction of property or an appalling loss of life.

Another consideration which, for storage dams, affects the choice of general location, rather than the exact position, is the quantity of silt carried by the stream. In some cases this is enormous, and may, in the course of a few years, completely fill the reservoir and destroy its usefulness. Sluices in the dam are never effective in preventing silting of the reservoir, except near the dam.

2. The Nature of Investigations. In considering a site for a dam, a preliminary investigation or reconnoissance must be made,

in order to determine whether or not the project is feasible. This may include considerable surveys, if a preliminary estimate of cost is necessary.

The final investigation is made after the project has become active, and includes all necessary studies up to and sometimes after the start of construction.

There are often many intermediate steps which partake of the nature of both of these divisions of the investigations, so that a clear distinction cannot be made. The extent to which the preliminary investigations should be carried will depend on the nature of the project and the information necessary for a complete report.

3. Preliminary Investigations. The preliminary examination of a dam site should be made with the following objects in view:

a. Advising as to the general adaptability of the site to the purpose of the project, and as to the desirability of proceeding with further investigations if other features are favorable;
b. Locating one or more possible sites for the dam;
c. Determining the character and extent of future investigation to be made;
d. Obtaining information on which to form a basis for a preliminary and approximate estimate of cost.

If results of instrumental surveys are not available, as is generally the case, the preliminary investigation should be as complete as it is possible to make it, without going into detailed surveys, as these will constitute a part of the final investigations which will follow if the preliminary report is favorable. The engineer should see every part of the proposed site or sites, make full notes, and take photographs or everything that may be of interest. Unless he is experienced in such work, he is likely to regret, when it is too late, that he did not record certain features which at the time of the investigation appeared to be of insufficient importance to warrant attention.

At each prospective site a study should be made of the materials of the foundation as far as it is possible to make such study from surface indications. If a foundation of rock is expected, the elevation of rock surface, the dip, direction, and character of the

strata and the probable quantity of overburden should be estimated. It is seldom that the exact character of the foundation can be ascertained from surface appearances, but sometimes a reasonable estimate can be made, especially if the foundation is to be of earth, or if rock is exposed throughout the width of the valley and the stream is small.

In case of a proposed important and expensive structure, where an accurate estimate of cost must be made, it will ultimately be necessary to complete the examination of the foundation by digging test pits and making borings, in order to ascertain the elevation and character of the rock. In his preliminary investigation, the engineer should decide, therefore, on the number and location of test pits and borings to be made later.

Possible locations for construction plant and camps should be sought, and their positions, estimated areas, and elevations recorded. The problem of unwatering the site should be carefully considered on the ground, and notes made as to the method to be adopted, the approximate depth of water, the estimated height of coffer-dams, and the effect of the velocity of the current and the nature of the foundation on their construction and the work of making them tight.

Specimens of materials available for construction and to be removed from the foundation should be obtained and preserved.

Rough estimates should also be made of the area and value of the land to be used, of the length and cost of railroads and public highways to be relocated, and of the area and cost of clearing and grubbing the reservoir, if required.

Search should be made for suitable stone, sand, gravel, timber, and other materials for construction purposes. The suitability of materials to be excavated from the foundations for use in construction should be considered. The quantity of timber required for coffer-dams and forms for concrete is generally so large that it is necessary to obtain this material from local sources of supply, if it is possible to do so. In cases where the work is of great magnitude, and very remote from the nearest shipping point, it has been found feasible to manufacture locally the cement for the masonry.

Search should be made for high-water marks, as an aid in estimating the magnitude of the maximum flood * and the prob-

* See Chapter XI.

able highest elevation of water below the dam, in order that its effect on the design and on the temporary and permanent works below the dam may be properly evaluated.

The accessibility of the site should be carefully investigated. The condition of existing highways, railroads, bridges, and navigable waters should be examined, having in mind the transportation of materials and supplies. If it is probable that the existing means of transportation are inadequate, or incapable of being made satisfactory, reconnoissance for one or more possible locations for a highway or railroad should be made.

The site should be studied for the purpose of arriving at a tentative conclusion as to the best type of dam to be built.

Limited surveys, including cross-sections of the valley, showing water surfaces, rock outcrops, and other information will be necessary if a preliminary rough estimate of cost is required.

4. Final Investigations. The final investigations are usually carried on under the supervision of the engineer who has conducted the preliminary examinations, or at least in accordance with his recommendations. The principal items are,

a. To determine the relative merits of two or more sites for the dam in question, so that a final location can be adopted;

b. To settle beyond a doubt the nature of the foundation, as affecting the safety and cost of the dam;

c. To fix the limits of the lands to be controlled for flowage, for the sites of structures, and for other necessary purposes;

d. To determine the length and character of relocation of railroads and public highways necessary on account of raising the water surface;

e. To ascertain the character of the Government regulations to be observed;

f. To obtain sufficient information for an accurate estimate of cost;

g. To fix the final location of the dam, construction equipment, camps, coffer-dams, construction highways, and railroads, as well as the probable source of materials of construction, and all other information needful to the constructing engineer;

h. To obtain all necessary information affecting the design of the dam.

Usually, there are not many sites to consider in the final examination. Of all that are available, an intelligent preliminary examination usually reduces the problem to a consideration of a very few, and, in many cases, where the problem is simple, it is sufficient for a final choice of site. Thus, a rigorous investigation of two or more sites is not always necessary in order to make a selection; in fact, such a requirement is the exception, rather than the rule. At any rate, it is never necessary to conduct the final investigation throughout its whole scope for more than one location, as the final choice of site can be made before the investigations have proceeded very far.

The foundation is one of the most important features to be investigated. If it is to be earth, a series of test pits or borings should be made, in order to determine the nature and extent of treatment which will be necessary to produce a stable and practically impervious footing. The investigations should include tests to determine the probable bearing power of the earth, or the number and length of bearing piles which will be necessary, and the nature and length of sheet-piling to be driven to prevent excessive leakage.

Except for low dams, the foundation should be rock. For rock the investigation should fix, not only the depth of the ledge below the surface, but its nature or suitability for a foundation. This is usually accomplished by test pits or wash-borings to rock surface and core drilling into the rock. With the latter it will be possible to determine, from the samples produced, not only the character of the rock, but the location of pervious or soft pockets, seams, and other faults, by the action of the drills during the boring operations. Water forced into the drill holes under pressure will indicate the perviousness of the foundation.*

Too much stress cannot be placed on the necessity of making extensive investigations of the foundations before construction begins, if an accurate indication of the construction difficulties, the cost of the work, and the tightness of the foundations is desired. A very good description of modern methods of conducting wash and core borings is to be found in Chester W. Smith's "Construction of Masonry Dams."†

* See Art. 61.

† McGraw-Hill Book Co., Inc., 1915.

The problem of determining the proper location and depth of borings and the interpretation of the results obtained is very difficult. It is impossible to give a general rule for the spacing and depth of borings, as each case presents a different problem. All probable zones of weakness, such as the plane of contact between intrusive igneous rock and rock of sedimentary origin should be thoroughly investigated.

It should be remembered that a valley, in the making, will usually be started at the weakest part of the geological formation. One is, therefore, likely to find at every dam site a geological reason for the particular location which the stream has adopted, and such indications will assist materially in the determination of the zones requiring more thorough investigation.

The spacings and depth of borings will be governed, not only by the geological formation, but also by the height of the dam, and the extent to which the importance of the work will justify the expense of the investigation. A careful study of the data obtained from the borings as they are drilled will influence the depth and location of additional borings.

In important work, particularly for high dams, the services of an expert geologist should be obtained, and his recommendations should be given full consideration. The danger of excessive leakage is not confined to the vicinity of the dam; such leakage may extend for considerable distances on each side of the dam site, and even through the ridge to an adjacent valley lying at a lower elevation.

Surveys should be made of the lands necessary for the site of the reservoir and of the dam, and for various other purposes. The results of these surveys should be indicated on maps of adequate scale, depending on the value of the lands to be obtained and, therefore the accuracy with which such information must be shown. These maps should indicate:

a. The original edge of the stream at low, ordinary, and high water;

b. The edge of the proposed reservoir at low, ordinary, and high water, usually with a sufficient number of intermediate contours to enable an estimate of the storage contents to be made;

c. The location of the dam, the sites for construction equipment, camps, and other proposed structures;

d. The property lines of the lands required. For the reservoir, a distance from the original shore line to the edge of the survey equal to two or three times the distance from the original to the new shore line will usually be sufficient. However, where part of the land required includes a large percentage of an individual estate, it may be necessary to purchase the entire tract. In this event the surveys should be extended to cover the total area.

e. The location of all railroads, public highways, bridges, dams, important structures, estates, enterprises, and other items of importance on and in the neighborhood of the proposed reservoir and dam site;

f. The proposed new location of railroads and public highways which must be relocated on account of raising the water surface, showing the boundaries of the lands required for such purpose;

g. A brief description of each parcel of land required should be placed on the maps, but a complete description should be given in a supplementary report, including the approximate value of lands, water rights, structures, etc., to be taken over.

Government requirements will include the observance of riparian rights, if the water is to be retained or diverted; the necessity of providing log chutes, fishways, or navigation locks; regulations covering the design and construction of dams, and allied matters. The Federal Government has jurisdiction over all navigable streams; also in most States it is required that the plans for the dam must receive the approval of the State Engineer or other official before construction starts.

A map of the dam site should be prepared showing,

a. Contours of the natural surface within and near the area to be occupied by the dam, construction equipment, camps, and other proposed structures. The required contour interval will be determined generally by the height of the dam. Usually, an interval of about 5 per cent of the height will be found to be sufficiently close.

b. The location of the river, existing and proposed structures, coffer-dams, test pits, borings, rock outcrops, high-water marks, proposed quarries, sand banks, and all other items of use to the designing and constructing engineers.

This map should be accompanied by cross-sections of the site at frequent intervals, showing the depth of test pits and borings, and the estimated elevations of all underground materials and their nature.

For estimating purposes, contour maps and profiles of the railroads and public highways to be relocated, as well as the railroad or highway for providing access to the site, may be necessary.

The results of the final investigations should be incorporated in a complete report, and this should be carefully preserved, as years may elapse between the time of making such investigations and the beginning of construction operations.

CHAPTER II

THE CHOICE OF TYPE OF DAM

5. General Considerations. It is not within the scope of this volume to describe in detail all the advantages and disadvantages of the different types of dams which have been built, but merely to outline briefly the adaptability of each type, in order that the reader may be able to comprehend the relative limitations of masonry dams.

The usual types of dams may be summarized as follows:

Solid gravity masonry dams,
Hollow gravity masonry dams,
Arched masonry dams,
Earth and rock embankments,
Timber dams, and
Other types.

Many combinations of these types have been constructed.

The choice of the type best suited to a particular location or use is a matter on which experienced engineers will often differ considerably, and is quite often purely a matter of judgment and experience. However, an intelligent study of the existing conditions and requirements will assist materially in the choice.

Safety, of course, is the first consideration. It is impossible to build with safety some types of dams under certain conditions of foundations and other characteristics of the site. Consideration of this question will often decrease considerably the number of possible types from which to choose.

The first cost of the structure, as affected by the availability and price of construction materials and other characteristics of the site, is, perhaps, of next importance.

The choice of type is often limited by the funds available for construction of the dam and other requirements of the project. It will sometimes be found that the difference in cost between an expensive, permanent dam and an inexpensive structure of short

life and high maintenance charges, if set aside at compound interest, will be more than sufficient to provide funds for the higher maintenance cost and a sinking fund to cover the rapid depreciation of the less expensive type. It may be said, however, that, in general, the most permanent dam will be found to be the most economical, and it is usually adopted for ordinary sites, unless the structure is for temporary use, or if sufficient funds are not available.

A comparison of the several types of dams follows:

6. Solid Gravity Masonry Dams.* There is no type of dam more permanent than one of solid masonry, nor does any other type require less for maintenance. It is adaptable to all localities except where a sufficiently impervious cut-off at and below the surface is impractical of attainment, where there is danger of considerable uplift, or where the low bearing strength of the foundation prohibits its use. It is imperative that high masonry dams be built on rock foundations. Low dams of this type are sometimes built on earth or piles, but such support, for dams more than about 30 ft. in height, should be adopted with caution.

The solid gravity masonry type, being the most common of all masonry dams, is the safest, according to the popular idea; and, in this respect, has an advantage when the enterprise is affected, to a large extent, by public opinion, as in municipal or other public works.

The difference in first cost between solid and hollow gravity masonry dams is the subject of considerable debate. The solid dam requires less cement per cubic yard of concrete, less form work, less expense in placing concrete, and has no steel reinforcement. On the other hand, the hollow dam requires considerably less concrete per linear foot of dam.† It is the author's opinion, based on a number of comparative estimates, that for a remote location, where materials of construction are expensive, the hollow type will usually cost less to build than the solid type; but, in an ordinary location, comparatively near a railroad, where there is a good quarry, and a sand bank is convenient, the reverse is true.

The solid gravity masonry dam will usually cost more than a timber dam. However, this may not be the case if a first-class,

* See Chapters VI and VII.

† Usually from 35 to 40 per cent of the concrete required for a solid dam.

rock-filled, timber crib dam is adopted at a site where a coffer-dam is required for its construction, and if timber is expensive.

An earth or rock embankment will almost always cost considerably less than any form of gravity masonry dam, if materials for the former are found convenient to the site. Therefore, if conditions admit of an embankment, that type of dam is usually to be preferred. The limitations of embankments will be mentioned later.

There is considerably less material in an arch dam than in any other masonry type, and consequently it will cost much less to construct. However, as will be pointed out later, a site suitable for an arched dam is the exception, rather than the rule.

7. Hollow Gravity Masonry Dams. Most hollow dams have been constructed of reinforced concrete of the types described in Chapter VIII.

Compared with most methods of construction, reinforced concrete is in its infancy. Although its durability has not been tested for as long a period as plain concrete, it has, thus far, shown itself to be as permanent as can be desired.

When the element of time is a governing consideration, the hollow dam possesses some advantage over the solid dam, because, there being less concrete to deposit, it can be constructed in a somewhat shorter period.

Turbines and other apparatus have often been placed within hollow dams, thereby making a saving in the necessary housing for such appliances.

A hollow dam is often adopted in preference to a solid gravity masonry dam in localities where considerable uplift * under the latter type would be expected. The hollow dam has a distinct advantage, owing to the fact that the narrow walls and buttresses are subjected to a practically negligible uplift pressure, the water under the dam having a direct exit.

Another advantage claimed for the hollow dam having an up-stream face with considerable batter, is that it is impossible for it to overturn; as the resultant of all forces, for any depth of water, falls well within the base. However, this advantage is more fanciful than practical, as either type, if properly designed, should give no cause for worry in that respect.

Hollow dams, being lighter per square foot of area covered,

* See Art. 15.

can, by having spread footings, be made to exert less unit pressure on the foundation than solid dams. For this reason the former type is sometimes adopted where the requisite support for a solid dam is lacking.

8. Arched Masonry Dams.* This type is adaptable when the length is small in proportion to the height, and when the sides of the valley are composed of good rock which can resist the end thrust. It is the ideal permanent dam, containing much less material than other masonry types, and being equally permanent, it is always adopted where conditions permit. Unfortunately, however, sites suitable for this type are seldom found.

The weight of the arched dam is not counted on to assist materially in the resistance of external loads. For this reason there is always sufficient weight of masonry to resist any possible uplift on the base.

Combinations of arch and solid gravity masonry dams are common for sites where the length is thought to be insufficient to permit of the adoption of the pure arched type. Such dams are designed to resist the loading by gravity, but are curved in plan and designed with a smaller margin of safety than if straight and with no arching possible.

9. Embankments. When plenty of materials are convenient to the side, embankments can usually be built for considerably less cost than any form of masonry gravity dam. The use of this type, however, is often limited by the necessity of providing a more suitable spillway for the passage of floods. It is not safe to allow water to spill directly over the embankment, even if it is well paved, unless the volume of the flood per linear foot of crest is small. Therefore a spillway of more suitable character is a necessary adjunct. In some instances such a spillway would require most, if not all, of the available length of the dam; in which case an embankment would be out of the question.

The quantity of seepage through pervious material is proportional to the distance the water is required to travel. An earthen embankment, having the longest base in proportion to the height, is particularly adaptable to sites having pervious foundations.

With proper maintenance, the embankment dam should be as permanent as the best. The necessary maintenance charges are comparatively high during the first year or two, but become rapidly

* See Chapter IX.

less as the structure settles into its final position and becomes well compacted, tight, and overgrown with proper vegetation to withstand wash from rains.

Earthen dams possess a distinct advantage in landscape work where it is desired to change as little as possible the appearance which Nature has given to the site.

10. Timber Dams. A timber dam is the ideal temporary type; although when well designed, constructed and maintained, it may last fifty years or more. Maintenance charges, however, are very high, compared with other types.

Timber dams are seldom very tight. In fact, a small leakage is necessary for the proper preservation of the timber. Such leakage, however, is of importance only when the value of the stored water is exceptionally high.

This type is often used on soft foundations where masonry dams are out of the question, as a slight settlement, which, in the former would be permissible, would, in the latter, be an element of considerable danger.

Owing to a scarcity of funds, a timber dam is sometimes adopted with the intention of utilizing it later as a part of the necessary coffer-dam for the construction of a more permanent structure.

11. Other Types. Various other types of dams have been designed and built. These include structural steel dams, peculiarly shaped masonry dams, the many forms of movable dams, and others. These, however, may be considered as either structures of unique character, suitable for special conditions not admitting of comparison in the general sense, or types which were the creation of fanciful engineers of radical tendencies.

CHAPTER III

FORCES ACTING ON DAMS

12. Nomenclature. The following nomenclature will apply, in general, to all parts of the text. Special nomenclature, applicable to arch dams, is given in Chapter IX. Unless definitely mentioned, all forces are stated in pounds and all dimensions in feet.

W = A *vertical force*; positive when directed downward;

P = A *horizontal* force; positive when directed toward the left;

P_i = Ice pressure per linear foot of dam;

R = A resultant of forces;

$\Sigma(W)$ = The algebraic summation of the vertical components of all forces acting on the dam above a given joint, including uplift, but excluding the reaction at the joint; positive when directed downward;

$\Sigma(P)$ = The algebraic summation of the horizontal components of all forces acting on the dam above a given joint, excluding the reaction at the joint; positive when directed toward the left;

$\Sigma(Wx)$ = The algebraic summation of the moments, about a given point, of the forces contained in the summation, $\Sigma(W)$; positive when counter-clockwise;

$\Sigma(Px)$ = The same for the summation $\Sigma(P)$;

A = An area, in square feet, or the area of a water-shed, in square miles;

a = The distance from the top of the dam to the water surface;

C = A constant;

c = The percentage of area of joints or base subjected to uplift;

e = The eccentricity of a loading (see "Irregular Bases," Art. 22, and Fig. 12);

E = A subscript used to represent the condition of empty reservoir;

r = a subscript used to represent the condition of full reservoir;

f' = The actual coefficient of static friction at a given joint or the base;

f = The coefficient of static friction of the same materials as indicated by well-dressed test specimens;

g = The acceleration of gravity = approximately 32.2;

h = A vertical distance, a height of masonry, a head of water, etc.;

h_c = The measured head on a spillway crest;

h_t = The total head on a spillway crest;

h_v = The head corresponding to a given velocity;

H = The total height of a dam above a given elevation;

I = The moment of inertia of a figure;

k = The percentage of voids in earth or silt, expressed as a decimal;

L = The top width of dam;

l_0 = The known length of a horizontal joint;

l = The unknown length of the horizontal joint next below;

l_t = The total length of a spillway crest;

l_n = The net, or effective, length of a spillway crest, Art. 43;

l_c = The average width of the channel of approach to a dam;

m = A distance to the right or left of the center of gravity of a figure; see definition given in "Irregular Bases," Art. 22;

N = A flood coefficient, Art. 65;

n = The number of complete end contractions on a spillway crest;

p = Unit pressure or compressive stress, in pounds per square foot;

p' = The same at the down-stream extremity of the base;

p'' = The same at the up-stream extremity of the base;
(The foregoing system of primes and double primes applies also the following special values of p.)

p_r = The unit *vertical reaction* * of the foundation at a joint or the base, exclusive of uplift pressure;

p_v = The unit *vertical compressive stress* * at a joint or the base, inclusive of uplift pressure;

* See foot-note, p. 43.

p_i = The unit *maximum inclined compressive stress* in the masonry or the foundation;

p_u = The unit *effective uplift* * on a joint or the base;

p_n = The unit *normal pressure* of water or earth on the face of the dam;

Q = The total quantity of water passing over the spillway crest, in cubic feet per second;

Q_m = The maximum flood from a given drainage area, likely to be exceeded only once in T years;

q = The quantity of water passing over each linear foot of effective spillway crest;

r = The radius of a circle;

S = A factor of safety, Art. 25;

t = A period of time, in seconds;

T = A period of time, in years;

u = The horizontal distance from the down-stream extremity of a joint to the point of intersection of the resultant, R, with that joint;

v = A velocity; in feet per second;

w = Unit weight;

w_1 = Unit weight of masonry;

w_2 = Unit weight of water;

w_3 = Unit weight of earth;

x = In general, a vertical distance. Also used as the lever arm of both vertical and horizontal forces;

y = The horizontal distance from an origin of moments to the up-stream extremity of a joint or the base;

z = The horizontal distance from an origin of moments to the point of intersection of the resultant, R, with a joint or the base;

α = The angle of repose of earth;

θ = The angle of inclination with the vertical,† of the resultant, R, of the forces, $\Sigma(W)$ and $\Sigma(P)$;

ϕ = The angle of inclination, with the vertical,* of the face of the dam;

ϕ' = The same for the down-stream face at a given elevation;

ϕ'' = The same for the up-stream face at a given elevation.

* See Art. 15 and foot-note, p. 45.

† This is the common definition, as, in general, the joints and bases are horizontal. For inclined joints or bases, the angles θ and ϕ should be measured from a normal to the joint or base.

13. General Considerations. The first consideration in designing a dam is the determination of the nature of the forces acting on the structure. These forces may be considered as consisting of the following:

a. Water pressure,
b. Earth pressure,
c. Atmospheric pressure,
d. Ice pressure,
e. Wind pressure,
f. Wave pressure,
g. Weight of the dam,
h. Weight of the foundation,
i. Reaction of the foundation.

The nature of most of these forces, unfortunately, is such that they do not admit of exact determination, and their amounts, direction and location must be adopted by the designer after a thorough consideration of all obtainable facts bearing on the case, and the exercise of his best judgment, based on his experience and that of others who have had to deal with similar problems.

It must always be borne in mind that conditions in no two dams are alike, and that a general theory must never be applied to a particular case without thought as to the possible need of modification to suit the conditions peculiar thereto.

14. External Water Pressure. The weight of a cubic foot of fresh water has been determined to be

62.42 lb. per cu. ft. at 32° F.
62.26 lb. per cu. ft. at 75° F.
62.00 lb. per cu. ft. at 100° F.

The weight usually adopted in the design of dams is 62.5 lb. per cu. ft.

The total pressure, P, of quiet water on any vertical submerged plane of area A is

$$P = w_2 A h_3, \quad \ldots \ldots \ldots \quad (1)$$

where w_2 is the weight of 1 cu. ft. of water and h_3 is the distance from the center of gravity of the plane to the surface of the water. The force, P, will be horizontal.

In Fig. 1, let 1–2 represent a submerged vertical rectangular plane of unity width, measured perpendicular to the paper, and having its top edge parallel to and a distance, h_1, from the surface of the water. As the width of the plane is unity, the length, 1–2 $=h$, will be a measure of its area, A.*

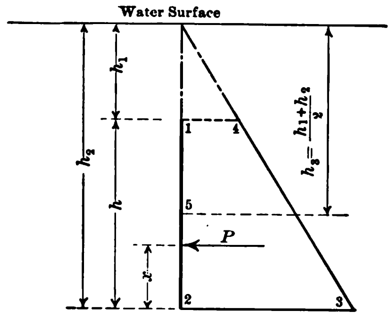


FIG. 1.

The center of gravity of the plane, 1–2, is at the point, 5, midway between 1 and 2.

The total pressure, P, on the plane, 1–2, may be obtained from Eq. (1):

$$P=w_2Ah_3=w_2h\frac{h_1+h_2}{2},$$

since A is of unity width.

Substituting the value, $h=h_2-h_1$, there results

$$P=\frac{w_2h_2^2}{2}-\frac{w_2h_1^2}{2}. \qquad \qquad (2)$$

The location of the force, P, may be determined in the following way: Lay off the horizontal line, 1–4, equal to w_2h_1, the unit water pressure at point 1. Lay off the line, 2–3, equal to w_2h_2, the unit water pressure at point 2. Then a horizontal intercept between any point on the line, 1–2, and the line, 4–3, will equal the unit water pressure at that point, and the total area, 1–2–3–4, will equal

* In the design of dams, it is customary to consider a slice bounded by two planes perpendicular to the axis of the dam and 1 ft. apart, thereby simplifying the calculations by allowing the neglect of the third dimension. Unless specifically stated, it should be assumed, in all that follows, that such a slice is being considered.

the total pressure, P. The force, P, will pass through the center of gravity of the area, 1–2–3–4, which is a vertical distance above point 2 equal to

$$x=\frac{3h_1h+h^2}{6h_1+3h}. \quad \text{(3)}$$

In the design of dams it is found convenient to deal with horizontal and vertical forces only. If the plane, 1–2, is inclined, as in Fig. 2, the resultant total pressure, R, on the plane may be resolved into horizontal and vertical components, P and W. The horizontal component, P, will be equal to the pressure on the projection, 2–5, of the plane, 1–2, and its amount and location can be calculated from Eqs. (2) and (3). The vertical component, W, will be equal to the weight of water above the plane, 1–2, namely, within the boundaries, 1–2–6–7. The force, W, will pass through the center of gravity of the figure, 1–2–6–7.

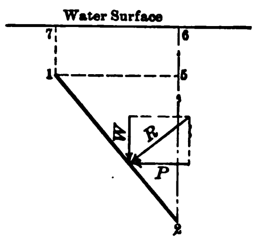


FIG. 2.

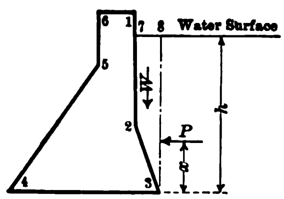


FIG. 3.

Inasmuch as it is possible for a dam to be entirely submerged, with water pressure on every square foot of surface, we will now investigate the forces due to water pressure in the following order:

a. On the up-stream face,
b. On the top,
c. On the down-stream face,
d. On the bottom.

In Fig. 3, the horizontal component, P, of the total water pressure on the up-stream face of the dam is equal to the total pressure on the plane, 3–8, as indicated by Eq. (2). In this case, h_1 being equal to zero, and h_2 being equal to h,

$$P=\frac{w_2h^2}{2}. \quad \text{(2a)}$$

The distance, x, from point, 3, to the force P, may be found from Eq. (3). For this case there results

$$x = \frac{h}{3}. \quad \quad (3a)$$

The vertical component, W, is equal to the weight of water within the area, 2–3–8–7, and passes through the center of gravity of that area.

In Fig. 4, the height, h, of the dam is not as great as the depth of water, h_2, so that the water is constantly spilling over the crest. In this case the dam will not be subjected to horizontal water

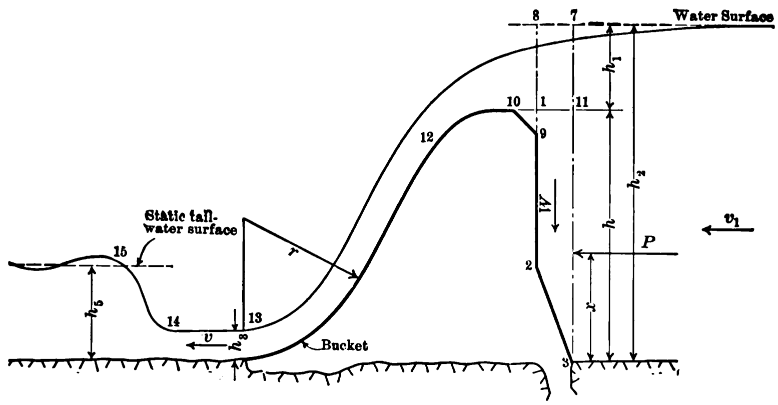


FIG. 4.

pressure on the plane, 1–8, above the crest. On account of the velocity of the water passing the plane, 8–9, which approaches very nearly spouting velocity in well-designed dams, the vertical component of the pressure on the plane, 9–10, and on the top of the dam is always neglected. To be on the safe side, however, the horizontal component of the pressure on the plane, 9–10, is included.

Considering, again, vertical and horizontal forces only, we have a vertical force, W, equal to the weight of water within the limits, 7–8–2–3, passing through the center of gravity of that area, and a total horizontal water pressure, P, equal to the pressure on the plane, 11–3, found by Eq. (2), and located by Eq. (3).

The heads, h_1 and h_2, should correspond to the measured head

on the dam; that is, to the water surface in the channel of approach sufficiently remote from the dam to be beyond reach of the surface curve.*

An additional head, equal to twice the velocity head of the water in the channel of approach, should be added to allow for impact on the face of the dam.†

If $v_1 = \frac{q}{h_2}$ is the velocity of approach, the average unit impact pressure is approximately

$$p = 2w_2h_v = 2\frac{w_2v_1^2}{2g} = \frac{w_2v_1^2}{g}, \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the total impact pressure on the dam is

$$P' = 2w_2h_vh = \frac{w_2v_1^2h}{g},$$

where g is the acceleration of gravity (about 32.2), q is the discharge per linear foot of crest, and w_2 is the unit weight of water.

Fig. 5 represents a dam containing a barrier, such as flashboards, projecting above the crest, or closed gates held between piers. In such cases, the weight of the water within the boundaries, 1–2–3–4, should be included in the calculations as well as the horizontal pressure on the plane, 1–2.

The stream of water indicated in Fig. 4, flowing down the face of the dam, is, on account of its velocity, considered as transmitting no pressure to the dam.

Pressure on the dam from water standing in the lower pool, as indicated in Fig. 5, may be treated in the same manner as described for water pressure on the up-stream face.

In the case of a spillway, tail-water does not always exert a pressure on the dam, as its depth adjacent to the dam is often practically eliminated by the impact of the water spilling over the crest.‡ This condition is indicated in Fig. 4.

Mr. A. H. Gibson,§ in his discussion of standing waves in non-

* See Art. 43.

† See Gibson's "Hydraulics," Art. 96; D. Van Nostrand Co., 1908.

‡ See Karl R. Kennison's paper "The Hydraulic Jump in Open Channel Flow at High Velocity," *Transactions*, Am. Soc. C. E., Vol. LXXX, p. 338.

§ Gibson's "Hydraulics and its Applications," Art. 86. J. Wiley & Sons, 1908.

uniform channels, states that the conditions under which this phenomenon will occur are

$$v^2 => gh_3,$$

and

$$h_5=\sqrt{\frac{2h_3v^2}{g}+\frac{h_3{}^2}{4}}-\frac{h_3}{2}.$$

For spillway dams, v^2 is always greater than gh_3, hence we may expect a reduction in depth of tail-water, provided h_5 is less than the value given in the second equation,

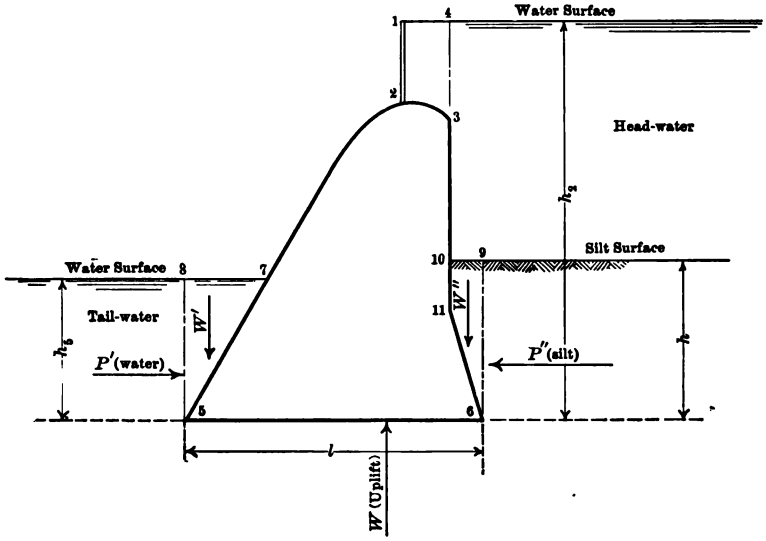


FIG. 5.

A more convenient form may be obtained by writing the second equation as follows:

$$h_5=\sqrt{\frac{2h_3{}^2v^2}{gh_3}+\frac{h_3{}^2}{4}}-\frac{h_3}{2}.$$

But $h_3{}^2v^2=q^2$, where q is the discharge, in cubic feet per second, per linear foot of crest. Making this substitution, there results

$$h_5=\sqrt{\frac{q^2}{16.1h_3}+\frac{h_3{}^2}{4}}-\frac{h_3}{2}. \quad \cdots\cdots \quad (5)$$

The thickness, h_3, of the sheet of falling water for any given head, h_1, and height of dam, h, may be found from Figs. 25 and 26. A method of determining the discharge, q, corresponding to the head, h_1, is given in Art. 43.

If h_5 is greater than the value given in Eq. (5), the water flowing over the spillway will expend its energy in creating eddies, and will cause no reduction in tail-water level.

If h_5 is equal to this value, a standing wave, 14–15, (Fig. 4), or "hydraulic jump," will occur at the toe of the dam.

If h_5 is less than this value, the standing wave will occur at a distance down-stream, where the depth, h_5, has increased an amount sufficient to fulfill the conditions of Eq. (5).

Therefore, if, in any example, it is found that h_5 is less than the value given in Eq. (5), the pressure of the water on the down-stream face of the dam may, theoretically, be neglected.

The foregoing has been based on theory alone. Although Gibson's equation has been fairly well substantiated by experiments on a small scale, the actual height, h_5, of the standing wave for large structures may differ considerably from that calculated. It is recommended, therefore, that when the depth of tail-water is within 20 per cent of the value given in Eq. (5), the pressure of tail-water should be considered doubtful and the dam tested for stability with and without it.

For a practical application of this theory see Art. 47.

Obstructions or "baffles," have often been placed in the tail-race, adjacent to the dam to prevent the standing wave from occurring below the apron and the swiftly moving water from having access to the unprotected portion of the foundation. Such an arrangement is indicated in Figs. 6 and 7. This feature will be further discussed in Art. 61.

15. Internal Water Pressure. Uplift. If the foundations are pervious, an upward pressure, or "uplift" will occur on the base of the dam, as indicated by the force, W, Fig. 5. The amount and location of this force depends on the relative perviousness of the foundation at various points and the details of the dam.*

In order to comprehend more clearly the principles of uplift under solid dams, consider a solid dam to be slightly raised from

* Such details refer to the difference between solid and hollow dams, and the use of drainage systems.

FIG 6.—Gatun Spillway Dam. Baffle Piers in Foreground.

FIG. 7.—Gatun Spillway Dam. Showing Effect of Baffle Piers.

its foundations, so that water flows through from the upper to the lower pool. The laws governing the pressure over the base are the same as those for the flow through pipes. In this case, neglecting the inappreciable loss at entrance, the pressure would diminish uniformly from w_2h_2 (Fig. 5), at point 6 to w_2h_5 at point 5. The total uplift would be

$$W = w_2 l \frac{h_2 + h_5}{2}.$$

The location of the force, W, can be obtained (for this as well as for other cases to be considered later), by locating the center of gravity of a force diagram constructed in the manner described for Fig. 1.

Assume, however, that although the dam is slightly raised, a portion of each square foot of the base has absolute contact with the foundation. The force, W, would then be reduced an amount in proportion to the percentage, c, of such area not in contact, Then,

$$W = cw_2 l \frac{h_2 + h_5}{2}. \qquad \ldots \ldots \ldots (6)$$

If there is an impervious barrier at point 5, the pressure over the portion of the base not in contact would be uniform and equal to w_2h_2, and the total uplift on the base would be

$$w = cw_2 l h_2$$

If the impervious barrier is at point 6, the uplift would be uniform and equal to

$$w = cw_2 l h_5$$

If the barrier is at any point between 5 and 6, the uniform unit pressure above the barrier would average cw_2h_2, and below the barrier cw_2h_5.

In practice, where the joint between the dam and the foundation, or other horizontal joints in the dam, are conducive to the passage of water, uplift will occur, and the foregoing laws will apply. The conditions are also the same for a horizontal seam in the rock below the base, with the exception that the points, 5 and 6, should be considered located at the entrance and exit of the seam, respectively.

Most engineers recognize the likelihood of uplift. The question of amount and distribution is, of course, impossible to determine exactly, and, of late years, this question has been the source of much debate. In order to bring the subject before the engineering profession for discussion, the late C. L. Harrison, on December 20, 1911, presented a paper before the American Society of Civil Engineers, in which he made some brief suggestions regarding the methods of estimating the amount and distribution of uplift on the foundations and horizontal joints of dams. The paper was quite freely discussed by some of the most prominent engineers. It is quite surprising to note the difference of opinion on the subject, varying practically from one extreme to the other.

The following are extracts from Mr. Harrison's paper * and his analysis of the discussion thereon:

For convenience in discussing this subject, reference is made particularly to masonry dams on rock foundations. The principles involved will apply equally to other foundations and to dams built of other materials. The upward pressure may be due to water getting into the foundation of the dam or into the dam itself.

Foundations vary so much in character that it is necessary to study each particular site before deciding to what extent water may get into them.

(1) In the case of a foundation of hard, sound rock, without either horizontal or vertical seams, there is no reason to expect that water will get into it and produce an upward pressure, and, in the design, no allowance should be made for it. In such cases the junction between the masonry and the foundation can easily be made water-tight.

(2) In the case where the foundation is stratified with well-defined horizontal seams, and the dam is located near a fall or rapids in the stream, so that the water may flow from the seams at the toe of the dam as freely as it enters them from the reservoir, the upward pressure will be approximately equal to the static head at the heel and gradually decrease to zero at the toe of the dam.

(3) Take a foundation similar to the foregoing in every respect except that the water in the seams of the rock cannot escape freely near the toe of the dam, but must flow some distance down-stream through rock or other materials before it reaches the surface of the ground, or must rise vertically to the surface. Then the upward pressure at the heel will be equal to the static head, and that at the toe will be equal to the head required to overcome the resistance to the water escaping at that point.

While these three cases present well-defined conditions, it is probable that at most sites the conditions will lie between those presented in Case 1 and Cases 2 and 3, that is, the water will not be in the foundation through-

* *Transactions*, Am. Soc. C. E., Vol. LXXV, pp. 142–225.

out its entire area, but will cover only a part of this area. This makes it necessary to study the foundation carefully at each site in order to determine to what extent water may get into it. When this upward pressure exists, weight must be added to the dam by additional masonry to counterbalance it. Generally, it will be found cheaper to make large expenditures to provide a cut-off in the foundation, which will not only reduce the uplift, but will also save the water. Such a cut-off should be located at the heel of the dam. If it is located under the middle of the dam, there would be an upward pressure under the up-stream half of the dam, due to the full head of the water in the reservoir.

* * * * * * *

In order to determine what allowance to make for pressures due to water which gets into the dam itself, one must first decide on the character of the construction. With suitable stone, sand, and cement, it is possible to build a masonry dam which will have no horizontal cracks or seams, and it is also possible to provide against vertical cracks, to a large extent, by expansion joints. Water in vertical cracks, however, does not produce an upward pressure. In such structures very little, if any, allowance should be made for the upward pressure due to water getting into the masonry.

If the materials for building water-tight masonry are not to be had at the site of the dam, and it is very expensive to import them, it is generally advisable to adopt a different class of masonry, which will probably be more pervious and also more difficult to construct without horizontal cracks or seams, thus allowing the water to enter the dam, and resulting in upward pressures. The extent of such pressures will depend on the character of the masonry and the care with which it is built, all of which must be known before an estimate can be made of the extent to which the water will get into the dam. The effect of this upward pressure, however, must be counteracted, either by increasing the section of the dam or by increasing its height above the water level in the reservoir, or by both. In many cases it may be advisable to provide drainage wells near the up-stream face to intercept the water and carry it off through pipes at the toe of the dam, thus reducing or eliminating its effects in the main body of the dam. After determining the type of masonry to be constructed, it is still a question of judgment, based on observation, tests, and experience, as to what the upward pressure in the dam will be.

* * * * * * *

An analysis of the discussion indicates the following conclusions:

1. For any stable dam the uplift in the foundation cannot act over the entire area of any horizontal seam, and in the masonry it cannot act over the entire area of any horizontal joint.

2. The intensity of uplift at the heel of the dam can never be more, and is generally less, than that due to the static head. Also, this uplift decreases in intensity from the heel to the toe of the dam, where it will be zero if the water escapes freely, and will be that due to the static head if the water is trapped.

3. The uplift in the foundation should be minimized by a cut-off wall,

under-drainage,* and grouting when applicable; and in the dam itself by using good materials and workmanship, and by drainage when advisable.

4. The design should be based on the conditions found to exist at each site after a thorough investigation by borings, test pits, and otherwise, and modified if found necessary after bed-rock is uncovered.

Mr. Harrison's paper should be read by all who are interested in the design of dams.

In practically all the designs of solid gravity dams embodying uplift which have come to the author's attention, the assumption has been a pressure varying uniformly from w_2h_2 (Fig. 5), at the heel to w_2h_5 at the toe, Eq. (6), being directly applicable. The only difference has been the percentage, c, of the base assumed to be subjected to the pressure. In Table I is given a list of some recently constructed high dams in the design of which extreme values of c were adopted.

TABLE I

PERCENTAGE OF AREA OF BASE SUBJECTED TO UPLIFT, ADOPTED FOR VARIOUS DAMS

Dam.	Value of c, Per Cent.
Wachusett	66
Cross River	66
Elephant Butte	33 (approximately)
Olive Bridge	66
Kensico	66
Lock Raven	66

All the dams mentioned in Table I contain drainage systems, cut-offs, and other features designed to reduce, as far as possible, uplift on the base. It must be remembered, however, that these dams are among the largest and most important structures built in this country, and failure would undoubtedly result in immense loss, not only at the dam but to other structures below it, and, in most cases, an appalling loss of life. For these reasons an unusually large margin of safety has been provided, not only in the assumptions of uplift, but in other features. Many dams, includ-

* Author's Note: Except for very high dams, comparatively few have been provided with under-drainage.

ing the large New Croton Dam of the New York City Water Supply system, contain no provision for uplift in the design. On the other hand, the failure of dams on very poor foundations, in a number of cases, may be attributed to insufficient provision for this force.

The author is of the opinion that a value of $c=\frac{2}{3}$ is the extreme limit for dams on fair rock foundations, and for the most important structures; and that ordinarily a much lower value may be adopted.

It is impossible to recommend definite values of c for the many classes of foundations which exist, no two of which are alike. The final choice must be made in accordance with the judgment and experience of the engineer after a thorough investigation of the site after the foundations have been exposed.

For earth foundations, a value of $c=1.0$ is necessary, but the distance which the water must travel may be considered as being the length of the base plus twice the depth of the impervious cut-off.*

Little, if any, uplift can exist in hollow dams of the types described in Chapter VIII. The pressure of water finding a passage through the deck or cut-off will be almost immediately relieved through the sides of the buttresses before it can penetrate any appreciable distance. The author knows of no case in which an assumption of uplift was used in the design of such structures.

Mr. Arthur P. Davis † gives the following general rules for the relative perviousness of several classes of foundations:

> The determination of the perviousness of natural formations is one of the most difficult things in Nature. Any examination of such formations which disturbs them changes the conditions which it is desired to know. For this reason, it is necessary to allow a large factor of safety in any estimates which involve this factor.
>
> In general, it may be said that water will more readily follow seams or bedding planes than devious paths through the material of the rock. It follows that it will pass more readily and in larger volume in the direction of stratification than in a direction normal thereto. Similarly, stratified rock will permit percolation more easily and in greater volume than good, massive rock, such as granite.
>
> Granular rock, such as sandstone, is likely to transmit more water through the rock itself than one of denser or finer grain, such as limestone or shale,

* See Art. 62.

† *Transactions*, Am. Soc. C. E., Vol. LXXV, page 208.

but no exact rule of this nature can be laid down, because there are many varieties of each kind of rock, with various percolating capacities. In general, however, the following rules may be taken as a rough guide:

1. Massive or crystalline rocks, such as granite, gneiss and schists, will transmit water less freely than those of sedimentary origin.
2. Stratified rocks will transmit water much more readily in the direction of stratification than transverse thereto.
3. In the direction normal to stratification, sandstone will generally transmit water more readily than limestone, and the latter more readily than shale.
4. Stratification on a plane approximately horizontal is the worst possible condition for introducing upward pressures beneath a dam. Conversely, the most favorable position in this respect for stratified rock is in vertical beds.

It is rational to assume that, in some cases, there may be less proportionate uplift on horizontal joints above the base than on the base. However, it is customary to adopt an uplift on all horizontal joints equal, proportionately, to that assumed for the base. This will lead to no appreciable error, and will greatly simplify the calculations, because, if the assumption of uplift for the joints were less than that used for the base, the upper part of the section designed would not suit those parts of the structure where the foundation is at a higher elevation.

The effect of silt deposits in reducing uplift pressure is discussed in Art. 16.

16. Earth Pressure. Practically all streams transport silt and gravel, particularly during floods, when, in some cases, the quantities are enormous. The construction of a dam across a stream results in a stretch of slack water which causes the material to deposit. It fills the upper end of the reservoir first, and gradually advances until it reaches the dam.

Quite often, sluices are constructed in the lower part of the dam, and, if periodically flushed in the proper manner, this limits the depth of such deposits adjacent to the dam.

An earth fill is sometimes deposited against the down-stream face of the dam, and may or may not be submerged.

The horizontal component of the silt or earth pressure, P'' (Fig. 5), exerted on the dam may be derived from Rankine's well-known equation:

$$P''=\frac{w_3h^2}{2}\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Where w_3 = the weight, in pounds per cubic foot, of the silt or earth, in air or submerged, as the case may be;
h = its depth, in feet;
and α = its angle of repose.

P'' is located a distance, $\frac{2h}{3}$, from the surface of the earth, and, when it is the result of submerged earth, it is in addition to the water pressure. The uncertain element of static friction between the earth and the dam is usually neglected, as it is on the side of safety.

When the face of the dam on which the earth or silt pressure acts is not vertical, as in Fig. 5, the weight of the material vertically above the plane (the area, 6–9–10–11), should be included in the forces acting on the dam, in the same manner as described for water pressure. As a matter of fact, the determination of the vertical and horizontal components of earth and silt pressures parallel exactly that described for water pressures, except that the portion of Eq. (7) in parentheses modifies the pressure in accordance with the internal resistance to movement, as measured by α, the angle of repose.

If w_3' is the weight of earth per cubic foot in air, and k is the percentage of voids, then in 1 cu. ft. of the fill there will be $(1-k)$ cubic feet of solids, weighing w_3' pounds. The weight of water displaced when this cubic foot of fill is submerged will be $w_2(1-k)$ pounds. Therefore, the weight, w_3, of the submerged fill will be

$$w_3 = w_3' - w_2(1-k). \quad \quad (8)$$

For example, if the material weighs 110 lb. per cu. ft. in air and has 30 per cent of voids, there will be (100–30 per cent) or 70 per cent of solids in each cubic foot. Therefore, 1 cu. ft. of the fill will displace 0.7 cu. ft. of water weighing $0.7 \times 62.5 = 43.75$ lb., and the weight per cubic foot of submerged fill will be $110 - 43.75 = 66.25$ lb. $= w_3$.

The weight, voids, and angle of repose vary, of course, with the nature of the material, and should be made the subject of investigation for each case.

Safe values often used for sand and gravel are $w_3' = 110$, $k = 30$ per cent, and $\alpha = 30°$. This corresponds to a value for w_3 of 66.2. For material capable of acting as liquid mud the usual

values are $w_3' = 125$, $k = 0$, and $\alpha = 0$. This corresponds to a value for w_3 of 62.5.

Thus it is seen that the usual unit weight of submerged earth is not far from that of water, but the angle of repose for use in Eq. (7) varies from zero for liquid mud to about 30° for sand and gravel.

It is not often that silt pressure on the up-stream face and uplift pressure from head-water are assumed to act simultaneously against the dam, unless the silt is of an unusually pervious nature. Ordinarily, the silt deposited by streams is an impalpable clay mixed with very fine sand or similar material, particularly impervious to the passage of water. It is the usual practice, where silt of an impervious nature is expected, to use in the calculations for the design that one of these two forces which has the greatest influence on the shape of the section.

It has been claimed that silt deposits relieve the water pressure on the up-stream face of the dam, but this is an assumption inconsistent with conservative design.

17. Atmospheric Pressure. Except as described later, atmospheric pressure is exerted on every square foot of the surface of

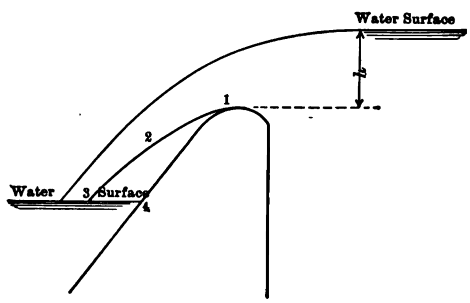


FIG. 8.

the dam. Fig. 8 represents a section of a spillway where the overflowing sheet of water is not in contact with the down-stream face of the dam.

In such cases the friction of the moving water surface, 1–2–3, entrains the air contained within the boundaries, 1–2–3–4, and carries it away. If the chamber, 1–2–3–4, has free access to the atmosphere, air will be easily supplied as fast as it is removed. It is not always feasible to provide such access, particularly if

the dam is very long and the head, h, on the crest is relatively great.

If the chamber, 1–2–3–4, does not have free access to the atmosphere, a partial vacuum or reduction of atmospheric pressure will occur therein. An adjustment of conditions, as indicated in Fig. 9, will then occur. The difference in atmospheric pressure on the two sides of the sheet will cause it to move toward the dam, the difference in pressure being balanced by the force overcome in changing the direction of the sheet.

The reduction of atmospheric pressure on the area, 3–4, will result in raising the water surface between the sheet and the dam an amount, h_1, sufficient to balance such reduction. The reduction of atmospheric pressure on the face, 1–4–4a, of the dam must be balanced by the stability of the structure. The resulting force,

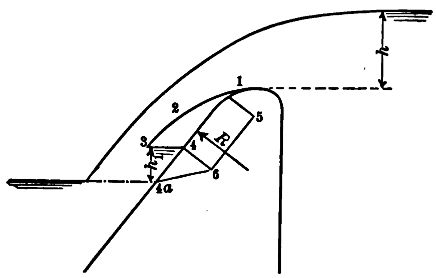


FIG. 9.

R, on the dam is equal to the trapezoid, 1–5–6–4a, in which the distances, 1–5 and 4–6, represent the reduction in atmospheric pressure, h_1, over the area, 1–4.

Quite often, in such cases, the reduction of atmospheric pressure becomes intermittent; that is, a partial vacuum within the chamber, 1–2–3–4, accumulates up to a certain amount, then a break occurs in the sheet admitting air, with a sudden return to normal pressure. The operation is repeated periodically. Such periods sometimes become of very short duration, causing a strong vibration, with a consequent tendency to loosen the dam from the foundation and thus decrease its stability considerably beyond that corresponding to the steady force, R.

As an example of the possible magnitude of such vibrations, the late J. P. Frizell * stated that the vibrations set up in this way

* "Water Power," by Joseph P. Frizell, 3d ed. J. Wiley & Sons, 1905.

by the water flowing over the old crib dam at Holyoke, Mass., rattled the windows in Springfield, six miles away.

On account of the impossibility of determining accurately the amount and effect of the force, R, it is customary to fit the downstream face of the dam to the lower nappe of the jet corresponding to the maximum head, h, which may occur on the crest of the dam. A method of determining the shape of the jet is given in Art. 42.

18. Ice Pressure. In common with other materials, ice expands and contracts with changes of temperature. In a reservoir completely covered with ice, a contraction due to a decided reduction in the temperature of the air will take place, opening up large cracks in the ice in which, subsequently, the water freezes solid. When the next rise of temperature occurs the ice expands, and if it is not free to slide up the banks of the reservoir, it will exert considerable pressure on the dam. This pressure usually causes the sheet of ice to buckle or crush. Should the conditions be favorable, the ice may exert an overturning force on the dam.

The thrust of ice is impossible of exact determination. It is, of course, limited to the crushing strength of ice which, however, is variously reported between 100 and 1000 lb. per sq. in. The latter value corresponds to the enormous amount of 144,000 lb. per sq. ft., and, where ice attains a thickness of 4 ft., amounts to the absurd value of more than a half a million pounds per linear foot of dam.

That ice thrust, under usual conditions, can never approach the latter value is proved by the fact that a great many dams are standing to-day which otherwise would certainly have failed. This, in part, may be explained by the fact that the full thrust of ice cannot be exerted on a dam:

1. If it is in a narrow gorge where the full effect of expansion cannot be felt;
2. If the opposite banks are sloping so as to allow the ice to ride up on them and thus relieve the pressure;
3. If, as has been claimed, the ice next to the dam cannot attain its full crushing strength owing to the greater temperature of the masonry.

Mr. C. L. Harrison's paper * on this subject brought out a

* *Transactions*, Am. Soc. C. E., Vol. LXXV, p. 142.

very free discussion. The following is quoted from his closing remarks in reply thereto:

It is generally agreed that this force [ice pressure] should be considered, and allowance made in some cases, but not in others. The limitations of these cases are not definitely stated by those who have discussed the subject. In order to give the matter more definite shape, it may be suggested that, under the following conditions, it is not necessary to provide for ice pressure:

1. For the ordinary storage reservoir with sloping banks, in climates where the maximum thickness of ice is 6 in. or less—for dams with southern exposure this limit may be placed as high as 1 ft. None of the discussions fixes this limit, but it is what the writer has in mind as a reasonable provision.

2. For reservoirs which are filled during the flood season and from which all the stored water is drawn off each year during the low-water season. This would include even the large reservoirs on the head-waters of the Mississippi River, where the ice has a thickness of more than 4 ft., and the atmospheric temperatures reach 50° below zero.

3. For storage reservoirs where the water will be drawn off each year during the winter to a level where the dam is strong enough to resist the ice pressure.

4. For reservoirs where the contour of the ground at the high-water level is such that the expansive force of the ice will not reach the dam.

* * * * * * *

The dams cited in Table II, where a value has been given to the ice pressure in the design, are in the vicinity of New York or Boston, where the maximum thickness of ice may be taken as about 2 ft. No allowance is made for ice pressure in the New Croton Dam, which is in the same climate.

TABLE II

ICE PRESSURE ADOPTED FOR VARIOUS DAMS

Dam.	Location.	Allowance for Ice Pressure, in Pounds per Linear Foot.
Wachusett	Boston	47,000
Olive Bridge	New York	47,000
Kensico	New York	47,000
Croton Falls	New York	30,000
Cross River	New York	24,000
New Croton	New York	No allowance

The stored waters in all the reservoirs inTable II are for domestic supply, and, excepting Olive Bridge and Kensico, are in service. The reasons given for the smaller allowances made for the Croton Falls and Cross River Dams

are that local conditions will prevent the full ice thrust from reaching the dams, and also that they are located up stream from the New Croton. If either of these dams should fail, no valuable property would be damaged, and the waters would flow into the New Croton Reservoir. Those responsible for the design of the New Croton Dam believed that no allowance should be made at this dam for ice thrust. At first glance, this looks like a wide range in judgment, but it must be remembered that the foregoing statement gives only a part of the facts, and to this must be added the local conditions and the service the dam is to render in each case before judgment is passed on the wisdom of the design.

Reservoirs for domestic supplies are generally drawn down during the ice period, and the greatest expansion of the ice occurs at the end of this period, thus applying the pressure at a point below high-water level, where the dam is strong enough to resist it. If, however, such reservoirs are to be at high-water level during, and especially at the end of, the ice period, at any time during their service, then the proper allowance should be made for ice pressure in the design. The daily fluctuations in the water level in the forebay at power dams will usually prevent the ice from freezing to the dam, which, therefore, will not be subjected to thrust caused by the expansion of the ice in the pool above the dam. In such cases, the proper course seems to be, not to reduce the allowance, but to omit it altogether. If, however, a storage reservoir is to be filled to the high-water level during the full ice period, at any time during the life of its service, then not a partial, but the full ice pressure should be allowed for in the design of the dam. It is contemplated that the Kensico Reservoir is to be kept at or near the high-water level at all times, and therefore will be subject to full ice pressure at high-water level; also, the Olive Bridge and Wachusett Dams may at intervals be subject to this pressure at high-water level. It is entirely possible that it would be proper to allow for ice pressure on a dam in a given locality and also proper to make no allowance for such pressure on another dam in the same locality, depending on the service each is to render.

The fact that so many dams have been designed and built without making a specific or separate allowance for ice thrust, and have for years stood the test of actual service without failing, is an indication that ice pressures may not be as great as sometimes thought, or that the factors of safety allowed for other purposes are sufficient to take this pressure. On the other hand in the cases mentioned in this discussion, there seems to be good and sufficient reason for allowing for ice pressure in the designs.

Ice floes are capable of exerting comparatively little pressure against a dam. If the velocity of the approaching water is high and the crest is not clear, the most that can be expected is a local thrust, which need not be very large, as the ice in such cases is always soft.

Possible ice thrust on the down-stream side of the dam is usually neglected in the calculations.

19. Wave Pressure. It is seldom necessary to consider the effect of waves on the stability of the dam. Stevenson's formula for the determination of the height of waves is:

$$h = 1.5\sqrt{F} + (2.5 - \sqrt[4]{F}), \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where h is the height, in feet, and F is the fetch, or straight length of clear water, in miles.

Unless the waves break, due to the relative shallowness of the water near the dam, very little impact results. Except for dams which are unusually small compared with the size of the reservoir the usual dimensions obtained from other considerations are ample to resist the effect of wave action.

20. The Weight of the Dam. The unit weight of masonry varies considerably, depending on the ingredients of which it is composed. Table III, giving weights of masonry, is taken from the "American Civil Engineer's Pocket Book." *

TABLE III

WEIGHT OF MASONRY IN POUNDS PER CUBIC FOOT

ASHLAR:	
Granite, syenite, gneiss	165
Limestone, marble	160
Sandstone	140
MORTAR RUBBLE:	
Granite, syenite, gneiss	155
Limestone, marble	150
Sandstone	130

Table IV gives the approximate average weight of plain Portland cement concrete. Although the weight of concrete is affected greatly by the materials of which it is composed, it is influenced very little by changing the proportions of the ingredients. A wet mixture will ordinarily result in a somewhat lighter concrete than if the ingredients are mixed dry and well compacted.

* By Mansfield Merriman. John Wiley & Sons.

TABLE IV

APPROXIMATE WEIGHT OF PLAIN CONCRETE AND COARSE INGREDIENTS, IN POUNDS PER CUBIC FOOT

Coarse Aggregate.	Coarse Aggregate (Solid).	Concrete.
Trap	175–200	150–160
Gravel		140–160
Granite, syenite, gneiss	150–195	135–160
Limestone, marble, quartz	155–175	140–145
Sandstone, bluestone	140–155	130–140

If the nature of the ingredients is known, the weight of dry concrete can be calculated approximately by use of Taylor and Thompson's * table of quantities of materials required for one cubic yard of concrete.

Assume trap rock to be used for the coarse aggregate, weighing 175 lb. per cu. ft. in the quarry and containing 45 per cent of voids when broken up with dust screened out. One cubic foot of crushed stone will then weigh $175\,(1-0.45)=96.3$ lb.

Assume the sand to weigh 90 lb. when dry, and that it has the characteristics assumed in Taylor and Thompson's table.

Cement usually weighs about 100 lb. per cu. ft.

From Taylor and Thompson's table, the weight of 1 cu. ft. of 1:3.6 concrete can be computed as follows, assuming a barrel of cement to contain 3.8 cu. ft. and to weigh $3.8\times100=380$ lb.

One cubic foot of concrete requires

$\frac{1.11}{27}$ bbl. of cement at 380 lb.	15.6 lb.
0.47 cu. ft. of sand at 90 lb.	42.3 lb.
0.94 cu. ft. of crushed stone at 96.3 lb.	90.5 lb.
Weight of 1 cu. ft. of concrete	148.4 lb.

If 20 per cent of plums are used, the weight of the cyclopean concrete masonry composed of this same material would be as follows.

* "Concrete, Plain and Reinforced."

One cubic foot of cyclopean concrete requires:

0.8 cu. ft. of plain concrete at 148.4 lb.....	118.7 lb.
0.2 cu. ft. of plums at 175 lb.............	35.0 lb.
Weight of 1 cu. ft. of cyclopean concrete...	153.7 lb.

This method of computing the weight of concrete is quite approximate, and adopted values equal to 95 per cent of computed values are not too conservative. In important work, an exact analysis of the materials or tests of the actual weight of concrete blocks, should be made.

21. The Weight of the Foundation. Dams have sometimes been tied down to the rock foundation in order to increase their resistance to overturning, sliding, or both. The arrangement has consisted of steel bars grouted into holes bored in the rock and extended into the dam near the up-stream face. This practice, however, has been severely criticised lately, on the ground that a satisfactory anchorage of the bars in the foundation is seldom possible.

22. The Reaction of the Foundation. *A. Rectangular Bases.* Let $\Sigma(W)$* (Fig. 10) be the resultant of all vertical forces acting on the dam above the foundation, including uplift, and $\Sigma(P)$ the resultant of all horizontal forces. The resultant, R, of $\Sigma(W)$ and $\Sigma(P)$ will represent the resultant of all forces.

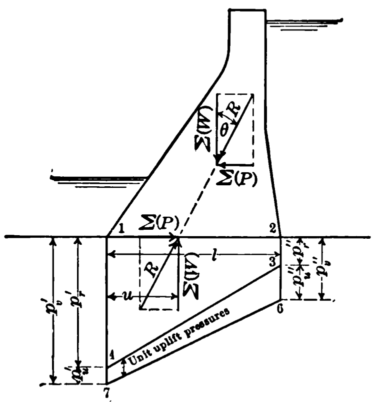


FIG. 10.

For the dam to be in static equilibrium, the resultant, R, must be balanced by an equal and opposite reaction, R, of the foundation, consisting of the total vertical reaction, $\Sigma(W)$, and the total horizontal shear or friction, $\Sigma(P)$.

The masonry of a dam and all classes of foundations, in common with other materials, are known to be elastic. The effect of

* $\Sigma(W)$ and $\Sigma(P)$ are here used to represent a general condition, and may be applied to either full or empty reservoir.

such elastic properties makes the absolute determination of the distribution of compressive stresses very difficult, if not impossible.

Because of the lack of knowledge on the subject, the distribution is assumed to follow a law of uniform variation, as in the design of beams, although it is known that, on account of such elastic properties, the unit pressures at the extremities of the base will be somewhat less than this method of distribution would indicate.

In Fig. 10, p_r' represents the unit vertical reaction* at point 1, and p_r'' that at point 2, due to the load, $\Sigma(W)$. An ordinate at any point from the base to the straight line, 3–4, will represent the unit vertical reaction at that point. The area, 1–2–3–4, will then represent the total vertical reaction, $\Sigma(W)$.

To derive an equation for the value of p_r', take moments about point 2. Then

$$\frac{p_r''l^2}{2}+(p_r'-p_r'')\frac{l^2}{3}=\Sigma(W)(l-u);$$

whence,

$$p_r'=\frac{3\Sigma(W)}{l}-\frac{3\Sigma(W)u}{l^2}-\frac{p_r''}{2}.$$

But,

$$\frac{p_r''+p_r'}{2}l=\Sigma(W),$$

or,

$$p_r''=\frac{2\Sigma(W)}{l}-p_r'.$$

Substituting this value of p_r'' in the second equation, there results

$$p_r'=\frac{2\Sigma(W)}{l}\left(2-\frac{3u}{l}\right). \quad \ldots \ldots \quad (10)$$

By taking moments about point 1, there may be derived, in like manner,

$$p_r''=\frac{2\Sigma(W)}{l}\left(\frac{3u}{l}-1\right). \quad \ldots \ldots \quad (11)$$

Dividing the base, 1–2, into three equal parts, as in Fig. 11, and calling the middle part the "middle third," it is seen from Eq.

* The reader should bear in mind the distinction between p_r, the *unit vertical reaction* due to load $\Sigma(W)$; and p_v, the *unit vertical compressive stress.* It will be shown later that $p_v=p_r+p_u$, as in Fig. 10.

(10) and (11) that when the resultant, R, intersects the base at the exact down-stream extremity of the middle third,

$$u=\frac{l}{3}, \quad p_r''=0, \quad \text{and} \quad p_r'=\frac{2\Sigma(W)}{l}. \quad . \quad . \quad . \quad (12)$$

When the resultant, R, intersects the base at the exact up-stream extremity of the middle third,

$$u=\frac{2l}{3}, \quad p_r'=0, \quad \text{and} \quad p_r''=\frac{2\Sigma(W)}{l}. \quad . \quad . \quad . \quad (13)$$

When the resultant, R, intersects the base at the middle point,

$$u=\frac{l}{2} \quad \text{and} \quad p_r''=p_r'=\frac{\Sigma(W)}{l}.$$

When the resultant falls outside the middle third, tension will exist at the opposite end of the base. This is indicated by substituting a value of u less than $\frac{l}{3}$ in Eq. (11), whereupon p_r'' becomes negative. Also, by substituting a value of u greater than $\frac{2l}{3}$ in Eq. (10), p_r' becomes negative. The former case is indicated in Fig. 11.

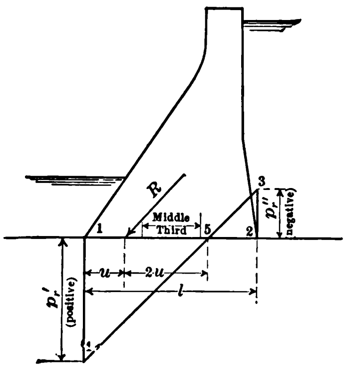


FIG. 11.

The unreliability of tension in masonry has led to the common practice of neglecting it as a possible factor in the stability of dams. Moreover, for reasons which will be mentioned later, the resultant for well-designed dams is always made to intersect the base within the middle third.

The values, p_r' and p_r'', derived from Eq. (10) to (13), inclusive, represent the unit *vertical reaction of the foundations on the dam*, corresponding to the load, $\Sigma(W)$. As $\Sigma(W)$ includes the force of uplift and as the water pressure causing uplift is exerted both upward and downward, the corresponding unit *vertical pressures*, p_v' and p_v'', on both the dam and the foundation, can be

obtained by adding to the calculated values of p_r' and p_r'' whatever effective * unit uplift has been assumed to exist. as indicated in Fig. 10.

To sum up:

For the resultant within the middle third:

$$p_v' = \frac{2\Sigma(W)}{l}\left(2 - \frac{3u}{l}\right) + p_u'. \quad . \quad . \quad . \quad . \quad (10a)$$

$$p_v'' = \frac{2\Sigma(W)}{l}\left(\frac{3u}{l} - 1\right) + p_u''. \quad . \quad . \quad . \quad . \quad (11a)$$

For the resultant at the down-stream extremity of the middle third:

$$p_v' = \frac{2\Sigma(W)}{l} + p_u', \quad . \quad . \quad . \quad . \quad . \quad (12a)$$

$$p_v'' = 0 + p_u''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12b)$$

For the resultant at the up-stream extremity of the middle third:

$$p_v'' = \frac{2\Sigma(W)}{l} + p_u'', \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

$$p_v' = 0 + p_u'. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13b)$$

The foregoing equations are applicable also to the vertical reactions and pressures on any horizontal plane above the foundation. They are simply modifications of the theory of flexure and direct stress as applied to horizontal rectangular sections.

B. Irregular Bases. If the base is not rectangular, as in the case of some hollow dams, the foregoing equations should be modified.

As indicated in Fig. 12, the total pressure at any point in the base will be the sum of the flexural stress at that point due to the eccentricity of the loading, a uniform direct compressive stress corresponding to the loading, and the uplift pressure.

From the principles of mechanics, the flexural stress at any point in the base is,

$$\text{Flexural stress} = \pm\frac{\Sigma(W)em}{I},$$

* p_u is termed "effective unit uplift" as it is the absolute uplift multiplied by the percentage, c, of the area over which it acts, as explained in Art. 15.

where m = any distance to the right or left of the center of gravity, in feet; positive on the side of the resultant, $\Sigma(W)$, and negative on the other side;

e = the eccentricity of the loading, in feet;

I = the moment of inertia of the base about an axis through its center of gravity perpendicular to the dam section, in feet units.

The stress will be positive (compression) on the side of the resultant

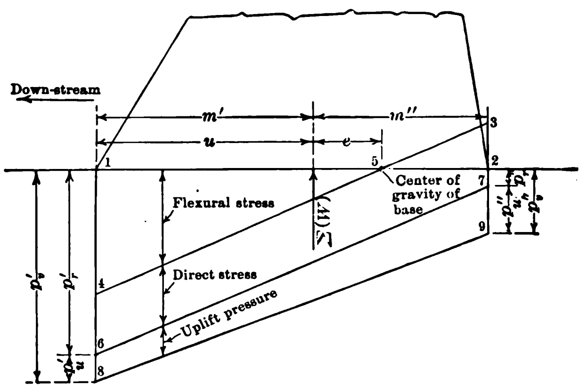


FIG. 12.

and negative (tension) on the other side, as indicated by the line, 4–5–3, in Fig. 12.

$$\text{The direct stress is equal to } \frac{\Sigma(W)}{A},$$

where A is the area of the base, in square feet. This stress is represented by ordinates between the lines, 4–5–3 and 6–7. Ordinates from the base, 1–2, to the line, 6–7, will represent the total flexure and direct stress, or the reaction of the foundation.

The unit effective uplift, if existing, is laid off below the line 6–7; the line, 8–9, finally represents the total unit pressure in the dam and the foundation.

If m' and m'' represent distances to the extremities of the base, as indicated, then

$$p_v' = +\frac{\Sigma(W)em'}{I} + \frac{\Sigma(W)}{A} + p_u', \quad . \quad . \quad . \quad . \quad (14)$$

and

$$p_v'' = -\frac{\Sigma(W)em''}{I} + \frac{\Sigma(W)}{A} + p_u''. \quad . \quad . \quad . \quad (15)$$

C. General. It is well to note here that the vertical unit pressures, p_v' and p_v'', are not the maximum compressive stresses in the foundation or the dam, but only the vertical components of such stresses. The determination of the maximum stresses will be taken up in Art. 33.

The distribution of the total horizontal shear, $\Sigma(P)$ (Fig. 10), is probably not possible of exact determination. Many attempts have been made to determine such distribution, with varied results.* Fortunately, it is not necessary to utilize the unit shearing stresses in order to determine the safety of the structure or to design the section. The amount of the total shear, $\Sigma(P)$, is sufficient.

* See Chapter VIII, "Masonry Dam Design," by Morrison and Brodie, 2d ed. John Wiley & Sons, Inc. 1916.

CHAPTER IV

REQUIREMENTS FOR STABILITY OF GRAVITY DAMS

23. Causes of Failure. There are two direct ways in which a dam will fail:

1. By sliding (*a*) on a horizontal joint above the foundation, (*b*) on the foundation, or (*c*), on a horizontal or nearly horizontal seam in the foundation.
2. By overturning (*a*) about the down-stream edge of a horizontal joint, (*b*) the base, or (*c*) a plane below the base.

The direct cause of sliding is the presence of horizontal forces greater than the combined shearing resistance of the joint or base and the static friction induced by the vertical forces.

The direct cause of overturning is the presence of horizontal forces, great enough, in comparison with the vertical forces, to cause the resultant, R (Fig. 10), of all forces acting on the dam above any horizontal plane, to intersect that plane outside of the limits of the dam.*

A dam may start to overturn, but finally fail by sliding. This is caused by the fact that, when overturning starts, not only is the shearing resistance at the plane of rupture materially reduced, but the admission of head-water pressure to the fissure at once reduces the effective weight of the dam, so that the frictional resistance due to the pressure of vertical forces is also diminished, and sliding ensues.

A dam with the resultant well inside the joint may overturn if the toe of the dam fails by crushing or other causes so as to reduce the effective length of the joint or base sufficiently to cause the resultant, R, to pass outside.

Theoretically, the dam extends indefinitely below the base, but, the area of the vertical section of the foundation being unlimited, stability below the base is usually assured. Failures have been caused, however, by an erosion of the foundation caused by

* Tension in the dam at the plane disregarded.

water spilling over the crest. In this event, one of two things may happen: the dam may become undermined so as to cause overturning, or the erosion may expose a horizontal seam filled with clay or other practically frictionless material, in which event, the rock ledge down-stream from the dam having been removed, it offers no resistance to sliding and the dam fails in that manner.

The gradual disintegration of the dam by weathering and other causes will, of course, finally result in its failure. Modern masonry dams, particularly if of concrete, are practically indestructible if well built and composed of proper ingredients. However, if proper precautions are not observed in the choice of materials and methods of construction, there may be rapid deterioration of the structure.

24. Rule 1. Governing the Location of the Resultant It is obvious that, if the resultant of all forces acting on a dam above any horizontal joint, including uplift, passes outside that joint, the dam will overturn unless the joint is capable of resisting tensile stresses. As the tensile strength is always disregarded as indeterminate and unreliable, we have as the first consideration that the resultant must intersect the joint. Further restrictions, however, are necessary.

It was shown in Art. 22, that, when the resultant falls outside the middle third, tensile stresses are set up at the other end of the joint. If the joint is incapable of resisting these tensile stresses, the elasticity of the masonry will cause a slight opening of the joint. Such an opening is particularly objectionable at the up-stream side when the pond is full, as it will admit full head-water pressure over the entire area not in compression, a condition considerably more severe than usually adopted for uplift. This additional uplift would result in a movement of the resultant toward the toe of the joint, with a further opening of the joint in tension, and a further increase in uplift. The progression may be sufficient to cause failure.

It is customary in most government requirements, and to a lesser degree in practice, to prohibit tension at the down-stream edge of the joints when the pond is empty. The logic of this requirement is open to debate, as one cannot imagine any dam of the usual type overturning up-stream before the water is let into the pond. Parker * states that if tension, reservoir empty, occurs

* "The Control of Water." D. Van Nostrand Co. 1913.

over 0.04 to 0.10 of the joint, it does not necessarily mean a bad design.

However, out of deference to precedent, and the usual government requirements, the author has adopted this condition.

We can now write the first designing rule.

RULE 1. GOVERNING THE LOCATION OF THE RESULTANT

Tension shall not exist in any joint of the dam, under any condition of loading. For dams with rectangular joints, the resultant of all forces acting on the dam above any horizontal joint (including uplift) shall, for full or empty reservoir, intersect the joint within the middle third.*

25. Rule 2. Governing the Inclination of the Resultant. The resultant, $\Sigma(P)$, (Fig. 10), of all the horizontal forces acting on the dam above any horizontal joint has a tendency to slide that part of the dam over the lower part. The shearing and frictional resistance of the joint must be sufficient to withstand this tendency.

The planes of weakness are the necessary horizontal joints (including the base), between two days' work. The shearing value of such joints, though sometimes considerable,† is unreliable. It is customary, therefore, to neglect any possible assistance from shear, and rely solely on the frictional resistance due to the vertical forces, $\Sigma(W)$, acting on the dam above the joint.

If f' represents the coefficient of static friction of the materials above and below the joint, then $f'\Sigma(W)$ will be the frictional resistance to sliding.

For equilibrium, $f'\Sigma(W)$ must be equal to or greater than $\Sigma(P)$. This may be expressed algebraically by:

$$f'\Sigma(W) = > \Sigma(P),$$

or,

$$\frac{\Sigma(P)}{\Sigma(W)} = \tan\theta = < f'.$$

* For joints which are not rectangular, as in a hollow dam, the resultant, in consideration of other requirements, usually falls so close to the center of gravity of the base as to obviate the possibility of tension. Where doubt exists, the joint should be tested for tension, as explained in Art. 22.

† W. A. Mitchell, in "Professional Memoirs," Corps of Engrs., U. S. A., Jan. and Feb., 1915, reports shearing strength on planes between new and old concrete amounting to nearly 50 lb. per square inch, as determined by tests.

where θ is the angle of inclination, with the vertical, of the resultant R (Fig. 10).

RULE 2. GOVERNING THE INCLINATION OF THE RESULTANT

The tangent of θ, the angle of inclination with the vertical, of the resultant of all forces (including uplift), acting on the dam above any horizontal plane, shall be less than the coefficient of friction at that plane.

The coefficient, f', in carefully constructed dams on rock foundations, with particular attention paid to obtaining very rough surfaces at the base and between two days' work, is usually considered to be at least twice as great as indicated by experiments on well-dressed specimens of the same materials. Therefore, if $\tan\theta$ is made equal to or less than the coefficient of friction, as indicated by such tests, a factor of safety in this respect of at least two will be provided; and the neglect of the probable considerable adhesion or shearing resistance at the joints and foundation will serve to increase further the factor of safety. The last equation can then be written:

For horizontal joints and rock foundations:

$$\frac{\Sigma(P)}{\Sigma(W)} = \tan\theta = < f, \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

where f is the coefficient of friction of the materials on each side of the joint or at the base, as indicated by tests on well-dressed specimens of the same material.

Values of f for masonry on masonry and masonry on good rock foundations have been assumed variously between 0.6 and 0.75. In general, and for careful work, a value of 0.75 is not excessive. Proper allowance, however, should always be made where the rock foundation is poor, or where it contains nearly horizontal seams close to the finished surface of the foundation. Such seams are particularly dangerous if they contain clay or other unstable material. The allowance to be made will depend on the character of the seam and its contents, its inclination, and the ability of the rock above the upper seam to resist movement.

On earth foundations, a large factor of safety should be provided, as the dam is as likely to slide on planes below the surface

as at the junction between the earth and the masonry. An artificial bond and increase in the coefficient of friction obviously cannot be obtained, as in the case of rock foundations. Eq. (16), therefore, should be rewritten for earth foundations as follows:

For earth foundations:

$$\frac{\Sigma(P)}{\Sigma(W)} = \tan \theta = \frac{f}{S}, \qquad \text{(16a)}$$

where S is the factor of safety desired.

For masonry dams on gravel, sand, and wet clay, approximate values of f are 0.50, 0.40, and 0.30, respectively. In conservative designs the dam is usually anchored to deep cut-off walls or piles, or a factor of safety, S, of 3 or more is adopted. The weight of the apron, which is an adjunct of every spillway dam on earth, will assist materially in reducing the tendency to slide.

26. Rule 3. Governing Compressive Stresses. Equations for the derivation of the vertical compressive stresses on the base and horizontal joints have been derived in Art. 22. It will be shown later that the maximum *vertical* compressive stresses are not the maximum stresses which occur in the structure. The maximum stresses occur on *inclined* planes, the vertical stresses being only their vertical components.

Many theories have been advanced for the determination of the maximum inclined stresses in dams; but, so far, no general agreement has been reached. The problem is not a determinate one, and, whatever theory is used, the solution is necessarily approximate.

For these reasons, it has been common practice to design dams with reference to maximum allowed vertical stresses, such stresses being chosen low enough, in the designers' judgment, to allow for the difference between them and the larger inclined stresses.

This method has resulted in the present type of solid high masonry dams having the down-stream face concave outward, and maximum inclined stresses which, in the lower part, are not constant at all elevations. To design a section in conformity with allowed maximum inclined stresses, in accordance with most of the theories advanced, would result in a section having a partly convex down-stream face, maximum vertical stresses which are not constant at all elevations, and different also in other minor respects.

Until the matter has received much needed attention by our engineering societies, and a general agreement is reached, it seems best to adhere to the present practice for shaping the section. However, a rational method of determining the maximum inclined stresses, no matter how approximate the assumptions on which it is based, is much more accurate than a pure guess. It is recommended, therefore, that, after the dam has been designed to conform to the allowed vertical stresses, the inclined stresses should be investigated in accordance with whatever theory is thought to be the best,* and changes made in the section if the inclined stresses are found to be too high. This course is all the more justifiable because the present method affects only the economy of the structure and not its safety, provided the inclined stresses are within allowed values.†

RULE 3. GOVERNING COMPRESSIVE STRESSES

The unit vertical and inclined compressive stresses in the dam and the foundation shall not exceed certain prescribed values.

The strength of masonry, as indicated by well-constructed test specimens, is not a satisfactory measure of its strength in large masses, owing to the less favorable conditions under which the latter is usually built. For this reason, as well as on account of the uncertainty in the determination of the maximum stresses, it is necessary to adopt working stresses for masonry dams corresponding to an unusually large factor of safety. It is, indeed, advisable to limit the stresses to values closely approximating that which precedent has shown to be safe.

* A combination of the theories of Enger and Bouvier, as described in Art. 33, is thought to give results more nearly correct than that of any of the others.

† This method is quite unsatisfactory, in that the stresses in the dam are limited; first, by vertical stresses fixed by usual practice according to the present method of design, and second, by inclined stresses which, it is thought, should receive equal attention, although seldom considered by designers. However, though the author does not deem it advisable to propose a rational change in the present method of design, without the consideration and discussion of the Engineering Profession, he cannot conscientiously countenance the acceptance of any design without due regard for the more important actual maximum or inclined stresses in the structure.

Table V indicates the maximum vertical and inclined stresses in several large existing dams, computed, in accordance with the theory indicated in Art. 33, from scaled values of the angle, ϕ, and from published records of computed maximum vertical stresses. These may be considered to be the greatest stresses which have thus far been adopted for conservative designs. The table also indicates the maximum stresses corresponding to examples of design of two authorities, and values recommended by others.

TABLE V

VARIOUS RECOMMENDED WORKING STRESSES AND EXISTING STRESSES IN MASONRY DAMS

Dam.	Vertical Stresses.		Inclined Stresses.	
	Toe.	Heel.	Toe.	Heel.
Kensico, above average rock	17,500	25,500	26,800	28,000
Kensico, maximum section *	31,000	32,000	43,700	35,300
New Croton	33,400	30,800	37,000	31,600
Estacada, Oregon (Hollow)	20,000	25,000	26,000	35,000
Wegmann's Practical Profile No. 3	16,800	21,000	46,000	25,400
Morrison and Brodie's Example of Design	28,000	31,000	52,400	31,400
Recommended by Parker	16,400	20,400	25,000	25,000
Recommended by Rankine	15,600	20,000		

* Stresses for the maximum section are probably not as great as computed, owing to the narrowness of the gorge between the sides of which the maximum section is located.

After an extensive study of published actual stresses computed for existing structures of many types, and the opinions of various authorities, it is recommended that, for maximum *inclined* compressive stress in masonry dams, a working value of one-ninth of the ultimate strength of the masonry should be adopted. This factor of safety should be sufficient to cover the element of uncertainty in the design and the relatively lower strength of masonry in large masses as compared with that of test specimens. Table VI, extracted from Taylor and Thompson's "Concrete Plain and Reinforced," indicates the ultimate strength of concrete of different proportions.

TABLE VI

RELATIVE ULTIMATE STRENGTH OF PORTLAND CEMENT CONCRETE, IN POUNDS PER SQUARE FOOT

Proportion.	Age One Month.	Age Six Months.
1 : 2 : 4	350,000	470,000
1 : 2½ : 5	310,000	420,000
1 : 3 : 6	280,000	380,000
1 : 4 : 8	230,000	300,000
1 : 5 : 10	180,000	250,000
1 : 6 : 12	150,000	200,000

For important structures, tests should always be made, at intervals, to determine whether the concrete, as being mixed, conforms in strength to the assumptions used in the design.

Although the ultimate strength of stone masonry has never been directly determined, a value of 250,000 to 300,000 lb. per sq. ft., for use as herein recommended, may be assumed as the ultimate strength of good rubble.

Until recent years, the maximum allowed unit *vertical* stresses commonly used for masonry dams on rock were those recommended by Rankine, for the dams of the City of Bombay, namely, 15,620 lb. per sq. ft. at the toe and 20,000 lb. at the heel. It will be shown later that, for constant vertical stresses, the amount of the inclined stresses in the masonry at either extremity of the base increases with the angle, ϕ, which that face makes with the vertical. Since, in the ordinary type of solid gravity masonry dams, the up-stream face is considerably less inclined than the down-stream face, Rankine reasoned that greater vertical stresses could be allowed at the heel than at the toe, and arbitrarily adopted these working vertical stresses in the dam on the design of which he was reporting.

The designers of recent dams, however, have exceeded these stresses considerably, as indicated by Table V. There is no logical basis on which to make a recommendation of working values of vertical stresses, because, as explained previously, the safety of the dam depends entirely on the amount of the existing inclined pressures which, for constant maximum vertical stress, varies with the shape of the section. It is necessary, however, to adopt

values which are within the limits of those commonly used for masonry dams, and which, moreover, will result in a section in which the maximum inclined stresses are not excessive. The following values will ordinarily fulfill both these conditions:

At the toe of solid dams $\frac{1}{21}$ of the ultimate strength.
At the heel of solid dams $\frac{1}{15}$ of the ultimate strength.
At the toe and heel of hollow dams $\frac{1}{15}$ of the ultimate strength.

For the foundations, it is universally agreed that bed-rock suitable for a high dam will in every case be stronger than the masonry placed upon it. For dams on earth foundations, the requirements of Rule 2 necessitate a very small angle of the resultant with the vertical. It will be shown later that the difference between the maximum inclined and the maximum vertical stresses in the foundation is a function of this angle and, therefore, so small as to be negligible. Common values for allowed stresses on earth foundations are as follows:

Clay..................	8,000 lb. per sq. ft.
Coarse sand...	4,000 to 8,000 lb. per sq. ft.
Fine silt.......	2,000 to 4,000 lb. per sq. ft.

It must be borne in mind that the qualities of such materials vary greatly, and that adopted values, in all cases, must be based on complete investigation at the site.

27. Rule 4. Governing Tension in Vertical Planes. Mr. L. W. Atcherly,* of London University, claims to have proved that, although the resultant, in dams of the usual type, may fall well inside the middle third of the base, there may still be considerable tension in vertical planes near the toe of the dam. Messrs. Morrison and Brodie † give an outline of Atcherly's theory, as well as the opinions of a number who have entered into a discussion of the paper. The paper attracted a great deal of attention at the time, and even influenced the design of several important structures. Later studies, however, have led to the general conclusion that such danger is not ordinarily to be feared. Atcherly's theory

* In a paper "On Some Disregarded Points in the Stability of Dams," See foot-note, p. 71.

† "Masonry Dam Design."

is based on an assumed distribution of horizontal shearing stresses, which remains to be demonstrated. A great number of existing dams, if investigated according to Atcherly's theory, would be found to be quite unsafe, if not incapable of sustaining their loads. Considering the hundreds of dams which have been conservatively designed and built without a thought of tension in vertical planes, not one of which is known to have failed through this source of weakness, it must be assumed that there is some error in Atcherly's methods, and that they impose too severe a condition on structures of the usual type.

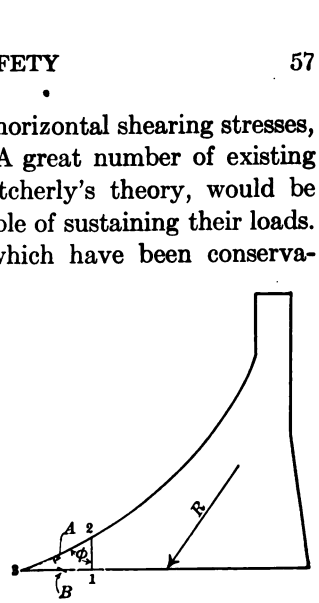


FIG. 12a.

However, one must not lose sight of the fact that tension will certainly exist in vertical planes if the inclination of the down-stream face of the dam is too great to transmit the loads to the foundation, as in Fig. 12a. In such cases there is danger of the toe cracking off on the line, 1–2, even though resistance to shear on that plane is ample.

The influence of these contingencies on the design of the dam is taken up in Art. 34.

RULE 4. GOVERNING TENSION IN VERTICAL PLANES

The inclination, with the vertical, of the down-stream faces of the dam shall be limited to prevent or safely withstand all possible tensile stresses in vertical planes.

28. Rule 5. Governing the Margin of Safety. The usual meaning of the expression "factor of safety," in the design of structures, is the ratio of the loading which will be just sufficient to cause damage or failure to the loading which has been adopted in the design. Factors of safety may be applied to the balance or resistance of a structure to overturning or to unit stresses. In a cantilever bridge it is usual to design the shore span so that it will have a resisting moment about the pier equal to twice the moment of maximum combined dead and live loads on the river end, thus providing a factor of safety of two against overturning. A steel tie-rod designed to be stressed to one-half its elastic limit will have

a factor of safety of two against damage (permanent elongation), and a factor of about four against failure.

In dealing with unit stresses, the term "factor of safety" is misleading, as it is usually applied to the ultimate strength, whereas in most cases, it is the elastic limit which is the limiting stress.

With regard to the balance of a structure, or the location of the resultant of forces, a factor of safety of one is, theoretically, all that is necessary, provided the loading is certain. However, on account of the usual uncertainty as to the loading, a factor of safety greater than one is necessary. In a dam, the head- and tail-water pressures for any given depth may be accurately calculated. A careful assumption of the weight of the dam should not vary more than 4 per cent from its correct value. Then we must adopt pressures for ice, silt, and uplift, which we are certain will not be exceeded, in order to justify a location of the resultant at the extremity of the middle third. If this is done the dam, if correctly designed, may be considered perfectly safe, provided, also, that the calculated induced unit compressive stresses amount to a safe proportion of the ultimate strength of the materials. The dam will have an additional element of safety on rock foundations, because the adhesion of the concrete to the foundation and in horizontal planes above the foundation, though certainly considerable, is always neglected. In fact, to this feature, alone, can be attributed the existence of a number of poorly designed dams.

RULE 5. GOVERNING THE MARGIN OF SAFETY

All assumptions of forces acting on the dam shall be unquestionably on the safe side; and all unit stresses adopted in the design shall provide an ample margin against rupture.

The margin of safety varies considerably in recent dams, and depends to a great extent on the use to which the structure is to be put, its magnitude, and the probable damage and loss of life which would result from failure. One of the most conservative designs of modern dams is that of the Olive Bridge Dam, of the New York City Water Supply System (Fig. 23). Probably few private enterprises could stand the burden of such conservatism.

29. Rule 6. Governing Details of Design and Methods of Construction. The shape of the section of the dam having been determined in accordance with the foregoing rules, careful atten-

tion must be paid to the details of the design and the methods of construction, in order that the structure may be satisfactory in every respect.

The location and extent of vertical building joints, passageways, and other planes of weakness must be well within proper limits, in order that the stresses used in the design will not be seriously increased. Such features as drains and cut-offs, on which the assumption of uplift has been based, must be carefully worked up, and other matters of much importance attended to. The engineer should be in a position to insure that the masonry in the structure will be of a quality to withstand safely the working stresses adopted in the design, practically water-tight, and durable. The final rule may be written as follows:

RULE 6. GOVERNING DETAILS OF DESIGN AND METHODS OF CONSTRUCTION

All details shall be adopted so as not to interfere with the assumptions used in the design; the masonry shall be of a quality suited to the working stresses adopted, and shall be practically water-tight and durable.

CHAPTER V

GENERAL EQUATIONS FOR DESIGN OF GRAVITY DAMS

30. General Considerations. It is impossible to derive a general equation for the direct determination of the shape of the section. The only possible solution is to design the dam, joint by joint, beginning at the top, making each joint conform to the designing rules given heretofore. Assuming the dam to have been designed from the top to a certain horizontal joint, 3–4, Fig. 13,

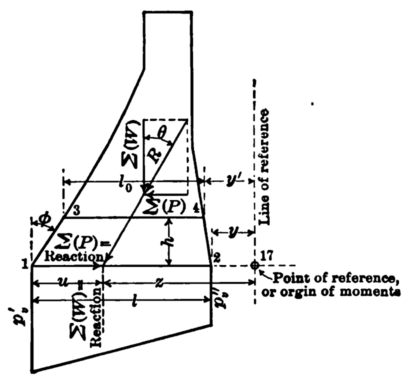


FIG. 13.

of length, l_0, equations can be written for the determination of the location, y, and length, l, of the next joint a distance, h, below, to conform to that one of the foregoing rules which happens to govern at the particular stage of the design.

The length of the joint will be mathematically exact, but the width of the section between the joints will be approximate only. By taking the vertical distance, h, small enough, say 15 per cent of the height of the dam above the joint considered, the resulting error will be negligible. The section will have polygonal faces, which, of course, may be smoothed up later, for appearance, with no appreciable change in the stability of the section.

Each step in the design is based on one of the rules which is

thought to govern. After each joint is calculated, and conforms to that rule, it is necessary to investigate the joint to determine if the other rules are also complied with. It is essential, therefore, for each designing rule, to derive two sets of equations, as follows:

1. "Equations of Determination" for each rule, with which to determine the length and location of successive joints in conformity with that rule;
2. "Equations of Investigation" for each rule, with which to investigate any pre-designed portion of the dam for conformity with that rule.

Such equations will be derived for each of the designing rules, it being found convenient to derive the latter set first in each case.

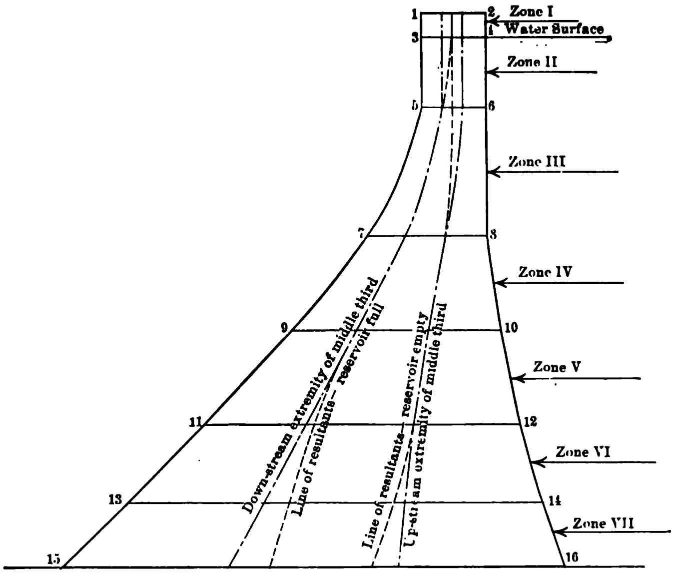


FIG. 14.

Before proceeding further, however, it seems desirable to indicate the influence of each designing rule on the general shape of the section of the dam. In order to do this, reference is made to Figs. 14 and 15, which indicate typical sections of a solid non-overflow and spillway dam, respectively.

The section of the dam may be divided into a number of zones, as indicated, it being necessary to design each zone in accordance with a different rule or combination of rules.*

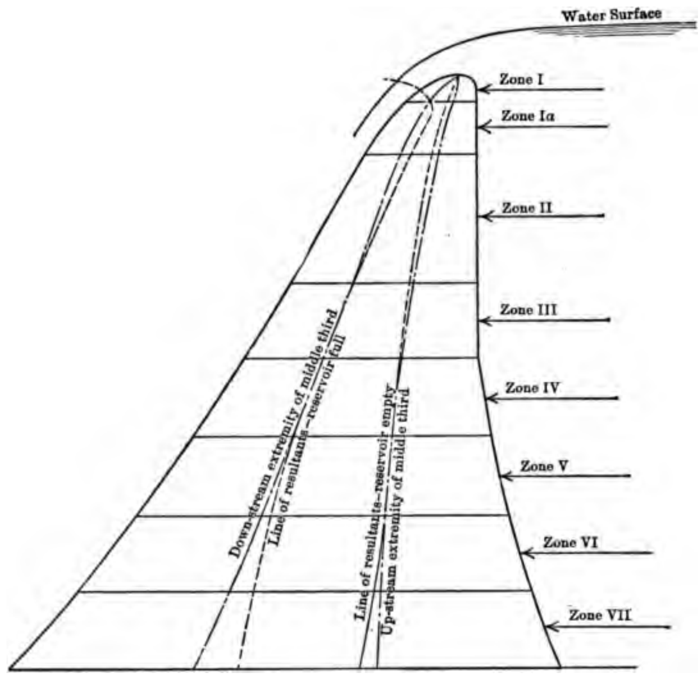


FIG. 15.

NON-OVERFLOW DAMS

When ice pressure occurs, the quantity of masonry in Zone I, above the water surface of non-overflow dams, is fixed by Rule 2, as sufficient weight must be provided to prevent the portion, 1–2–3–4, from sliding.

In Zone II the resultants, reservoir full and empty, lie well within the middle third, due to the fact that the width of the top is always greater than necessary to conform to Rule 1. Both up- and down-stream faces, therefore, will remain vertical † until,

* As far as the author is aware, Wegmann was the first to use similar divisions of the section of the dam to explain the methods of design.

† It is generally recognized that there is no economy in battering the faces of the dam unless that is necessary in order to conform to one of the designing rules.

at the plane, 5–6, the resultant, reservoir full, intersects the joint at the exact extremity of the middle third.

At the top of Zone III, the down-stream face must begin to batter in order to accord with Rule 1, reservoir full; and the resultant, reservoir empty, still being within the middle third, the up-stream face remains vertical until at the plane, 7–8, the resultant, reservoir empty, also intersects at the extremity of the middle third.

Therefore, at the plane, 7–8, the up-stream face must begin to batter, in order to accord with Rule 1, reservoir empty. In Zone IV the resultants, reservoir full and empty, intersect the joint at the extremities of the middle third.

The upper limit of Zone V is fixed by the condition of limiting vertical pressures, Rule 3. Usually, the maximum allowed unit pressure is reached at the down-stream face first. Below the plane, 9–10, the length of the joints must be determined by Rule 3, for full reservoir, and by Rule 1, for empty reservoir. This will result, for Zone V, in the resultant, reservoir full, intersecting well within the middle third, and, for reservoir empty, the resultant continues to intersect at the extremity of the middle third.

The vertical pressures at the up-stream face, however, will gradually increase, and, at the upper extremity of Zone VI will just reach the allowed working value. In Zone VI, therefore, the length of the joints will be determined entirely by Rule 3, the resultants, reservoir full and empty, intersecting well within the middle third.

As the section increases in height, the batters of both up-stream and down-stream faces increase. The down-stream face, in solid gravity dams, always has a flatter slope than the up-stream face. Consequently, at some elevation, such as 13–14, the inclination of the down-stream face from the vertical may reach the maximum allowed value. Zone VII, therefore, represents a portion of the dam in which the inclination of the down-stream face is apt to be greater than the limit fixed by Rule 4. It is unfortunate for the peace of mind of the designer if this happens, for, in that event, as will be indicated later, it will be necessary for him to start the design all over again, and provide a different arrangement in the upper part which will result in a steeper down-stream face.

SPILLWAY DAMS

Owing to a deficiency in the weight of the top of all spillway dams, the resultant, reservoir full, within Zone I, will necessarily fall outside the limits of the middle third, in direct violation of Rule 1. This condition must be met by special construction, as explained later. At the extreme top of the dam, where the weight of the masonry is negligible in comparison with the water pressure, the resultant will intersect at a distance infinitely remote, as indicated by the trend of the line of resultants, reservoir full, shown in Fig. 15.

For the same reason, above the bottom of Zone I*a*, the inclination of the resultant, reservoir full, will make an angle with the vertical greater than the allowed value, in direct violation of Rule 2. At the extreme top of the dam, where the weight is negligible in comparison with the water pressure, there will be no frictional resistance and sliding of the upper part of the dam must be prevented by shearing resistance.

It is obvious that, above the bottom of Zone I*a*, these two rules must be violated, as it is not feasible to supply enough masonry without obstructing the water spilling over the crest. Fortunately, however, the conditions are not severe and can be easily met by special treatment.

The conditions fixing the limits of Zones II to VII, inclusive, for spillway dams are exactly as previously described for non-overflow sections.

It should be borne in mind that the arrangement of zones indicated in Figs. 14 and 15 represent the usual conditions met in the design of solid dams. In hollow dams the arrangement will be different, but the general theory will apply. The equations for design given in the following articles will be derived with respect to solid dams; but they are equally applicable, in general, to hollow dams.

31. Equations for Rule 1. *Equations of Investigation.* To derive a general equation for the location of the intersection of the resultant, R (Fig. 13), with the joint, 1–2:

Let $\Sigma(W)$ represents the algebraic summation of the *vertical* components of all forces acting on the dam above the joint, 1–2, including uplift; $\Sigma(W)$, also rep-

resents the equal and opposite *vertical* reaction of the foundation;

$\Sigma(Wx)$ represent the algebraic summation of the moments about any convenient joint, 17,* of the forces above the joint, contained in the summation, $\Sigma(W)$; positive when counter-clockwise;

$\Sigma(P)$ represent the algebraic summation of the horizontal components of all forces acting on the dam above the joint, 1–2; $\Sigma(P)$ also represents the equal and opposite horizontal reaction of the foundation;

$\Sigma(Px)$ represent the algebraic summation of the moments about the point, 17, of the forces above the joint, contained in the summation $\Sigma(P)$; positive when counter-clockwise;

Subscript $_E$ represents the condition of empty reservoir;

Subscript $_F$ represents the condition of full reservoir;

Other symbols as indicated in Fig. 13.

The moment of all forces, acting on the dam, above the joint, about the point 17, is $\Sigma(Wx)+\Sigma(Px)$. The moment of the reactions about the same point is $\Sigma(W)z$, the moment of the force $\Sigma(P)$ being zero. For equilibrium, these moments must be equal; therefore, for the general case of either full or empty reservoir, we have:

$$\Sigma(Wx)+\Sigma(Px)=\Sigma(W)z. \qquad (17)$$

Solving for z, there results:

$$z=\frac{\Sigma(Wx)+\Sigma(Px)}{\Sigma(W)}. \qquad (18)$$

According to Rule 1, the distance, $z-y$, must be less than $\frac{2l}{3}$ for full reservoir and greater than $\frac{l}{3}$ for empty reservoir.

The usual forces acting under the condition of "full reservoir" are indicated in Fig. 16. Equations for the determination

* Point 17 should be on any convenient line of reference, and at the elevation of the joint considered.

of the amounts and locations of these forces have been given in Chapter III.

For "empty" reservoir, the force due to the weight of masonry is usually considered to be the only one acting on the dam. This is consistent with the loading after completion, but before the coffer-dam enclosure is allowed to fill, and before earth or silt is deposited against the dam. It usually gives the severest condition, as far as Rules 1 and 3 are concerned.

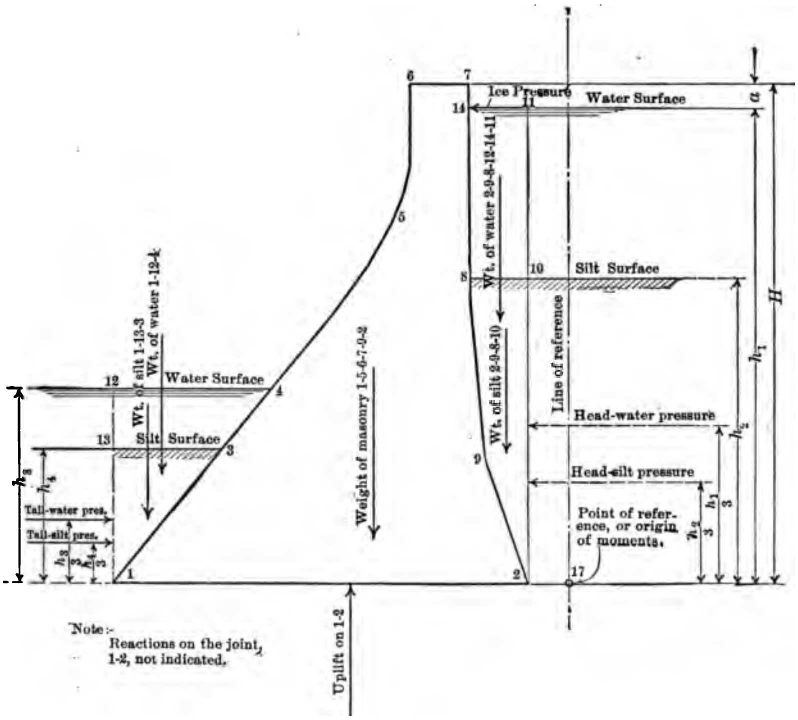


FIG. 16.

Equations of Determination. First. The general case, where the location of the resultant for both full and empty reservoir governs the design of the section, as in Zone IV (Fig. 14).

In Fig. 13, suppose the dam to have been designed from the top down to the joint, 3–4, and that it is desired to determine the length, l, of the next joint, 1–2, and its location, y, relative to the convenient point of reference, 17.

For full reservoir we may substitute $(y+\frac{2}{3}l)$ for z in Eq. (18). Adding the subscript, $_F$, there results:

$$y+\frac{2}{3}l=\frac{\Sigma(Wx)_F+\Sigma(Px)_F}{\Sigma(W)_F}. \quad . \quad . \quad . \quad . \quad . \quad (19)$$

For empty reservoir, by substituting $y+\frac{1}{3}l$ for z in Eq. (18) and using the subscript, $_E$, there results:

$$y+\frac{l}{3}=\frac{\Sigma(Wx)_E+\Sigma(Px)_E}{\Sigma(W)_E}. \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Thus we have two equations with two unknowns, y and l. The factors in the second terms of these equations, however, are functions of y and l. By substituting in the second terms the values of the various forces and moments, in terms of y and l, exact equations may be derived from which the values of y and l can be solved directly. Unless, however, the vertical components of water and silt pressure on both faces of the dam are neglected, such equations will be too intricate for practical application.* A sufficiently accurate tentative method of solving for y and l may be used, as follows:

From the general trend of the slopes of the up- and downstream faces, tentative approximate values of y and l may be used for the determination of the factors in the second terms of the equations. If the height, h (Fig. 13), is not more than 15 per cent of the height of the dam above the joint considered, the resulting calculated values of y and l will generally lie within 0.5 per cent of the truth, so that usually, a second trial substitution will be unnecessary.

* Messrs. Morrison and Brodie, in their "Masonry Dam Design," by neglecting the items mentioned, have derived such equations applicable to Rules 1 and 3 and combinations thereof. The author has found, however, by actual comparison, that his tentative method, hereinafter described, is sufficiently accurate, involves no more labor, and has the advantage of not only including all forces acting on the dam, but requires a tabulation of forces, moments, and other features in such a manner as to allow comparatively little opportunity for omissions or misapplications.

Parker, in his "Control of Water," has also derived similar equations but states: "I am not aware that these equations have ever been applied in practice, and as the result of actual experiment, I am inclined to believe that a return to first principles and trial and error is probably more rapid."

Second. The special case where the location of the resultant, reservoir full, is the only governing condition, as in Zone III. For this case the up-stream face is vertical and, therefore y, in Eq. (19), is known. For this reason, and also because the resultant, reservoir empty, is well within the middle third, Eq. (20), is not necessary, and the lengths, l, of successive joints in this zone may be solved from Eq. (19) alone.

Third. The special case where it is required to determine the elevation of the joint where the resultant, reservoir full, first intersects the down-stream extremity of the middle third, as at the bottom of Zone II. For most types of dams it will be found convenient to determine this elevation by trial, using Eq. (18). For

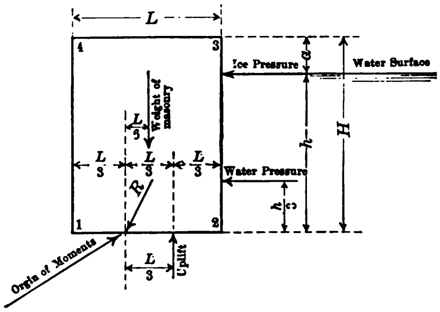


FIG. 17.

the non-overflow dam, however, it is possible to derive an exact equation applicable to the usual conditions of loading, namely, the external forces acting on the dam consisting of head-water pressure, ice pressure, and uplift.

In Fig. 17 let 1–2–3–4 represent the top of such a dam. Let it be required to determine the distance, h, below the water surface, at which the resultant, R, reservoir full, lies just at the extremity of the middle third.

The forces assumed to act on this portion of the dam are indicated in Fig. 17, and may be expressed as shown below. The origin of moments in this case is taken at the down-stream extremity of the middle third. Remember that counter-clockwise moments are considered positive.

Item.	*Force.*	*Moment.*
Vertical forces:		
Weight of masonry,	$+w_1LH$	$-\frac{w_1L^2H}{6}$
Uplift,*	$-\frac{cw_2hL}{2}$	$+\frac{cw_2hL^2}{6}$
	$\Sigma(W)=w_1LH-\frac{cw_2hL}{2}$;	$\Sigma(Wx)=-\frac{w_1L^2H}{6}+\frac{cw_2hL^2}{6}$
Horizontal forces:		
Ice pressure,	$+P_i$	$+P_ih$
Water pressure,	$+\frac{w_2h^2}{2}$	$+\frac{w_2h^3}{6}$
	$\Sigma(P)=P_i+\frac{w_2h^2}{2}$	$\Sigma(Px)=P_ih+\frac{w_2h^3}{6}$

Eq. (18) may now be written:

$$z=0=\frac{\Sigma(Wx)+\Sigma(Px)}{\Sigma(W)}=\frac{-\frac{w_1L^2H}{6}+\frac{cw_2hL^2}{6}+P_ih+\frac{w_2h^3}{6}}{w_1LH-\frac{cw_2hL}{2}}.$$

Substituting $h+a$ for H, there results:

$$(cw_2L^2+6P_i-w_1L^2)h+w_2h^3=w_1L^2a; \quad . \quad . \quad . \quad (21)$$

from which the value of h may be derived by proper substitutions.

Fourth. The special case in which it is required to determine the elevation of the joint, where the resultant, reservoir empty, first intersects the up-stream extremity of the middle third, as at the bottom of Zone III. No equation can be written for this case. It must be solved by trial, using Eq. (18).

Fifth. The special case where the location of the resultant, reservoir empty, and limiting pressures at the down-stream extremity of the joint (Rule 3), reservoir full, governs the design, as in Zone V. For this case, the resultant, reservoir full, being well within the middle third, Eq. (19), should be disregarded and Eq. (20) used in conjunction with another equation embodying the

* The uplift is assumed to vary uniformly from a value, cw_2h, at the up-stream end of the joint to zero at the down-stream end.

condition of limiting toe pressures. This combination will be taken up in the derivation of equations for compressive stresses.

32. Equations for Rule 2. *Equations of Investigation.* The tangent of the angle, θ, of the inclination, with the vertical, of the resultant, R, Fig. 12, is evidently

$$\tan \theta = \frac{\Sigma(P)}{\Sigma(W)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

According to Rule 2, $\tan \theta$, for rock foundations, must be equal to or less than the coefficient of friction, f; and for earth foundations, equal to or less than $\frac{f}{s}$.

Equations of Determination. As mentioned heretofore, it is necessary, in non-overflow dams, to provide sufficient masonry above the water surface to counteract the sliding force of ice. It is not practicable to write an equation for this condition, for, if the width of the top and the superelevation adopted on account of other considerations is not sufficient for this purpose, there is nothing to indicate whether an increase in the top width or an increase in the superelevation would provide more economy for the dam as a whole. A slight increase in either has little influence on the quantity of masonry in the dam, because an addition of material at this place will effect a greater or less reduction in the lower parts. The choice must be made in accordance with the judgment of the designer.

In spillway dams it is not practicable to provide sufficient masonry at the top to conform to Rule 2, to provide for ice thrust. In this case it is necessary to violate the requirements of Rule 2 and compensate by some special construction, such as keys, inclined joints, or monolithic concrete.* Violations of this kind are also sometimes found in the tops of concrete non-overflow dams, where it is possible to provide monolithic concrete within the danger zone.

Except as mentioned previously, it seldom happens that Rule 2 is a factor in the design of solid dams on rock foundations. Tan θ should always be determined, however, and compared with the allowed value, after each step in the design has been completed. If, at any stage of the design, it is found that $\tan \theta$ is greater than the value allowed by Rule 2, then several courses of procedure are

* See Example No. 4 of Art. 46.

open to the designer. It is evident, in such a contingency, that the fault lies in the lack of weight, or, in other words, lack of resultant vertical forces, $\Sigma(W)$. Additional weight may be obtained by adding to the top width, increasing the surcharge, increasing the batter of the up-stream or down-stream face, or a combination of these expedients. Each case must be decided on its own characteristics. It will usually be found most economical to increase the batter of the up-stream face, as this will include an additional vertical component of water pressure.

One of the principal reasons for the flat slope of the up-stream face of a hollow dam is the need for the large vertical component of water pressure to fulfill the conditions of Rule 2. A hollow dam with a vertical up-stream face would contain, on account of this rule, nearly as much masonry as a solid dam. The batter of the up-stream face of a hollow dam, if fixed by Rule 2, is found by successive trials, using Eq. (22).

33. Equations for Rule 3. *Equations of Investigation.* In Art. 22, the following general equations were derived for the determination of the maximum *vertical compressive stress* in the horizontal joints and base:*

At the toe,

$$p_v' = \frac{2\Sigma(W)}{l}\left(2-\frac{3u}{l}\right)+p_u'. \quad . \quad . \quad . \quad . \quad (10a)$$

At the heel,

$$p_v'' = \frac{2\Sigma(W)}{l}\left(\frac{3u}{l}-1\right)+p_u''. \quad . \quad . \quad . \quad . \quad (11a)$$

The symbols are explained in Art. 12 and indicated in Fig. 10.

However, as explained in Art. 26, the vertical compressive stresses are not the maximum stresses in the structure, the latter occurring on planes which are not horizontal. The question of the maximum, or "inclined," stresses in dams has been the subject of considerable discussion,† more particularly abroad than in

* Eqs. (12*a*) to (13*b*) inclusive, being for special cases, have been omitted for brevity, and may be easily included by the reader.

† L. W. Atcherly, Dept. of Applied Mechanics, University College, University of London, Draper's Co. Research Memoirs, Technical Series II; Sir Benjamin Baker, *Minutes of Proceedings*, Inst. C. E., Vol. 162, p. 120; L. W. Atcherly, *Engineering*, Vol. 79, p. 414; W. C. Unwin, *Engineering*, Vol. 79, pp. 513, 593, and 825; Ottley and Brightmore, *Minutes of Proceedings*, Inst. C. E., Vol. 172, p. 89; Wilson and Gore, *Minutes of Proceedings*, Inst. C.E., Vol. 172, p. 107; E. P. Hill, *Minutes of Proceedings*, Inst. C. E., Vol. 172, p. 134; William Cain, *Transactions*, Am. Soc. C. E., Vol. 64, p. 208.

America. It is unfortunate, however, that no general agreement has been reached on this subject.

The most commonly accepted theory is that of Enger, as expanded by Cain, Hill and others. The result ot Enger's theory is an inclined stress at the extremity of the joint equal to

$$p_t = p_v \sec {}^2\phi.$$

where p_v is the vertical stress and ϕ the angle of inclination of the face of the dam with the vertical. If a normal pressure, p_n, of water or silt acting on the face of the dam, is included, this equation will reduce to

$$p_t = (p_v \sec {}^2\phi - p_n \tan {}^2\phi) \text{ or } p_n.$$

In the last equation, the two values of p_t correspond to the two principal planes of stress; the governing condition being, of course, the greater of the two.

Enger's theory, however, is not generally applicable, because the whole argument is based solely on a triangular dam, with vertical up-stream face, loaded with water pressure level with the top. The author has found that the theory will not apply to any other shape of dam or loading; and that, for cases where the angle, θ, of the resultant, R, is greater than ϕ, p_t, from the foregoing equation, will give results too low. It is, indeed, rational to assume that for all values of θ greater than ϕ, the theory of Bouvier * will apply more closely. This theory gives as the value of the inclined stress,

$$p_t = p_v \sec^2 \theta.$$

This equation is probably the most accurate of any for the actual maximum stress in the foundation for all conditions.

Combining the two theories, the author proposes the following general equations for the true maximum, or *maximum inclined compressive stresses* in the dam and the foundation.

For maximum stress in the dam:

At the toe

$$p_t' = (p_v' \sec {}^2\phi' - p_n' \tan {}^2\phi') \text{ or } p_n' \text{ or } p_v' \sec {}^2\theta. \quad . \quad (23a)$$

* "Calculs de Résistance des Grands Barrages en Maçonnerie." *Annales des Ponts et Chaussées*, Aug., 1875.

At the heel,

$$p_i'' = (p_v'' \sec^2\phi'' - p_n'' \tan^2\phi'') \text{ or } p_n'' \text{ or } p_v'' \sec^2\theta. \qquad (23b)$$

For maximum stress in the foundation:
At the toe,

$$p_i' = p_v' \sec^2\theta. \qquad (24a)$$

At the heel,

$$p_i'' = p_v'' \sec^2\theta. \qquad (24b)$$

In these equations the general expression ($p_v \sec^2 \phi - p_n \tan^2 \phi$), gives the approximate flexural stress on planes, A, Fig. 12*a*, lying close to the point, 3, and normal to the face, 2–3. The value, p_n, is, of course, the normal pressure of external forces on the plane, 2–3, close to the point, 3. The general expression, $p_v \sec^2 \theta$, gives the stress between the dam and the foundation, on planes, B, lying normal to the resultant, R.

Eq. (23*b*) will not apply to the heel of hollow dams, if the horizontal area of the deck is included in the area of the base (as explained in Art. 49), the equation applying only to solid sections. Fortunately, however, it is known that, in all ordinary types of hollow dams having sloping up-stream faces, the maximum stress at the heel corresponds to the bearing of the deck on the buttresses. For practical application to hollow dams, see Art. 50.

Although the foregoing equations and the theories on which they are based are not exact, and, for certain conditions, probably not closely approximate, nevertheless the equations certainly indicate a value for the maximum stress considerably nearer the truth than the vertical pressure, p_v, generally used.

The author does not propose a change in the present practice, in which the shape of the section is fixed by certain allowed maximum vertical pressures, p_v, as indicated by Eqs. (10*a* and 11*a*). It is recommended, however, that the completed section should always be tested for maximum stresses in accordance with his proposed Eqs. (23) and (24), and altered if found necessary.

Equations of Determination. It has been pointed out that it is usual to consider only vertical pressures in the determination of the shape of the section, the section afterward being investigated for maximum stresses and changed if found necessary.

First. When the condition of limiting vertical toe pressures, reservoir full, and the condition requiring the resultant, reservoir

empty, to intersect at the up-stream extremity of the middle third governs the design, as in Zone V. This is a combination of Rules 1 and 3.

The location of the resultant is given by Eq. (18), from which, by using the subscript, $_F$, to represent full reservoir, there results,

$$z_F = \frac{\Sigma(Wx)_F + \Sigma(Px)_F}{\Sigma(W)_F}.$$

The distance, u_F, Fig. 13, is:

$$u_F = l + y - z_F = l + y - \frac{\Sigma(Wx)_F + \Sigma(Px)_F}{\Sigma(W)_F}.$$

The maximum vertical compression at the joint for full reservoir is at the toe of the joint, and is given by Eq. (10*a*). Transposing that equation, using the subscript, $_F$, and substituting for u_F the value just given, there results:

$$\frac{4\Sigma(W)_F l - p_e' l^2 + p_u' l^2}{6\Sigma(W)_F} = l + y - \frac{\Sigma(Wx)_F + \Sigma(Px)_F}{\Sigma(W)_F}.$$

Solving for l, there results:

$$\frac{p_u' - p_e'}{6} l^2 - \frac{\Sigma(W)_F}{3} l = \Sigma(W)_F y - \{\Sigma(Wx)_F + \Sigma(Px)_F\}. \quad . \quad (25)$$

For empty reservoir, Eq. (20) applies:

$$y + \frac{l}{3} = \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E}. \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Thus we have two equations, (25) and (20), with two unknowns, y and l, from which the location and length of the joint may be determined. The values, $\Sigma(W)$, $\Sigma(Px)$, and $\Sigma(Wx)$, for both full and empty reservoir, are also functions of y and l. Here, as in the equations of determination (first case), applicable to Rule 1, these values, in terms of y and l, may be substituted in Eqs. (25) and (20) and exact equations derived. However, as in the former case, certain omissions must be made in order to provide equations of practical application.* The author, therefore, proposes the

* See foot-note on page 67.

same method of trial substitution of approximate values of $\Sigma(W)$, (ΣPx), and $\Sigma(Wx)$, derived by anticipating, from the general trend of the faces of the dam, the location, y, and the length, l. If such substitutions are made in Eqs. (25) and (20) the resulting calculated values of y and l will be, after the first or second trial, sufficiently accurate for all practical purposes.

Second. When the conditions of limiting pressures, reservoir full and empty, govern the design, as in Zone VI.

The location of the resultant is given by Eq. (18), from which, by using the subscript, $_E$, to represent empty reservoir, there results:

$$z_E = \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E}.$$

The distance, u_E, is:

$$u_E = l + y - z_E = l + y - \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E}.$$

The maximum vertical compression at the joint, for empty reservoir, occurs at the heel, and is given by Eq. (11a). Transposing this equation, using the subscript $_E$, and substituting for u_E, the value just given, there results:

$$\frac{p_v''l^2 + 2\Sigma(W)_E l - p_u''l^2}{6\Sigma(W)_E} = l + y - \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E}.$$

Solving for l, there results

$$\frac{p_v'' - p_u''}{6}l^2 - \frac{2\Sigma(W)_E}{3}l = \Sigma(W)_E y - \{\Sigma(Wx)_E + \Sigma(Px)_E\}. \quad (26)$$

For full reservoir, Eq. (25) applies:

$$\frac{p_u' - p_v'}{6}l^2 - \frac{\Sigma(W)_F}{3}l = \Sigma(W)_F y - \{\Sigma(Wx)_F + \Sigma(Px)_F\}. \quad . \quad (25)$$

Here we have two equations with two unknowns, y and l, from which the location and length of the joint can be determined by the author's method of trial substitution, as in the preceding case, the discussion of which also applies here.

34. Equations for Rule 4. It was explained in Art. 27 that tensile stresses in vertical planes may be caused by an excessive inclination of the down-stream face with the vertical. It is seldom

that the shape of the section of the dam is fixed in any way by Rule 4. However, where this rule is a governing feature, there is no way of proportioning the shape of the section except by the cut-and-try method.

It is obvious that the direct cause of such tension at the toe is a relatively greater movement of the triangle, 1–2–3 (Fig. 12*a*), toward the left than that of the rest of the dam. This condition may be brought about by failure of the frictional and shearing resistances of the plane, 1–3, or by unequal inclined settlement of the foundation. The allowed inclination, ϕ', of the face, therefore, must be a function of the coefficient of friction, f' (shear on 1–3 being neglected, as usual), and the settlement coefficient of the foundation. The author knows of no satisfactory theoretical analysis of this feature. His empirical equations, based on safe approximate assumptions, are as follows·

For earth or pile foundations,

$$\tan \phi' = < \sqrt{\frac{10}{H}}, \quad \ldots \ldots \ldots \ldots \quad (27)$$

where H is the height of the dam.

For rock foundations,

$$\left.\begin{array}{l} \tan \phi' = < \frac{4}{3} f \\ \text{or} \\ \tan \phi' = < \sqrt{\frac{10}{H}} \end{array}\right\} \left.\begin{array}{l}\text{whichever allows} \\ \text{the greater value}\end{array}\right\}, \quad \ldots \quad (28)$$

where f is the allowed coefficient of friction.

The value ($\tan \phi' = < \frac{4}{3} f$) in Eq. (28) embodies the condition of no tension in vertical planes, but Eq. (27) and the second part of Eq. (28) embodies the condition of tensile stresses so small as to be negligible.

Eq. (28), for good rock foundations, does not impose a severe condition. Substituting a usual value of $f = 0.75$, there results $\phi' = 45°$, which is about the limit reached in high existing dams. According to the second part of Eq. (28), a value of ϕ' greater than 45° would be allowed in dams less than 10 ft. high.

35. Equations for Rule 5. Rule 5, governing the margin of safety, is applicable only to the determination of constant assump-

tions used throughout the design, and does not require equations for its correct application.

36. Equations for Rule 6. It is evident that no universal rules or equations can be written to provide against defective details and methods of construction. Details vary considerably with each dam, and depend mostly on the judgment of the designer to guard against infringement of the designing rules.

CHAPTER VI

THE DESIGN OF SOLID NON-OVERFLOW GRAVITY DAMS

37. General Considerations. As it will always be found convenient to start design at the top of the dam, the shape of the top of the section is the first consideration.

A superelevation of the top above high-water surface is sometimes desirable in order to get beyond the reach of waves,* for appearance, and for other incidental purposes. When ice pressure is assumed to act at the level of high water, some superelevation will be found necessary in order to fulfill the conditions of Rule 2 at the point where the ice pressure is applied. In any case it is probable that a superelevation of about 5 per cent of the height of solid non-overflow dams is, in general, productive of economy, rather than an expenditure of material, although sufficient proof has not been presented to verify this for all cases. However, there is no evidence to show that the adoption of a slight superelevation is uneconomical.

The most economical width of top of a solid non-overflow gravity dam is a direct function of the height of the dam, and is dependent on the assumptions used in the design. For dams of fairly uniform height, and designed in accordance with the usual designing assumptions, the most economical top may be assumed at about 14 per cent of the maximum height,† but for dams of considerable variation in height this figure should be somewhat reduced. It is usually considered that a value of 10 per cent is about the minimum advisable. The width of top of low dams is usually somewhat greater than that dictated by economy, as a roadway or passageway is often necessary, as well as sufficient width to withstand the shock of floating bodies. The details of the top of the dam having been determined, the up-stream and down-stream faces must be designed to conform strictly to the designing rules hereinbefore given, it being remembered that there

* For height of waves see Art. 19.

† See "The Economical Top Width of Non-overflow Dams," by the Author, *Transactions*, Am. Soc. C. E., Vol. LXXX, p. 723.

is no economy in battering either face unless necessary to conform to such rules. The determination of the shape of the section is simply a practical application of the general equations for design previously derived. In order to explain the application of such equations to solid non-overflow dams, the following examples are given:

38. Example No. 1. 200 ft. Solid Non-overflow Dam (Fig. 21). Assumptions:

w_1 = weight of masonry = 145 lb. per cu. ft.;

w_2 = weight of water = 62.5 lb. per cu. ft.;

p_v' = maximum allowed vertical compressive stress at the toe = 18,000 lb. per sq. ft.;

p_v'' = maximum allowed vertical compressive stress at the heel = 25,000 lb. per sq. ft.;

p_i = maximum allowed inclined compressive stress, in the dam and foundation, at the toe or heel = 42,000 lb. per sq. ft.;

H = maximum height of dam = 200 ft.;

c = the area of joints and base subjected to uplift = 50 per cent. The uplift is assumed to vary uniformly from head-water pressure at the heel to zero at the toe (there being, for this example, no tail-water);

f = the working value of the coefficient of friction of the joints and base = 0.75;

L = the width of top = 12 per cent of the height = 24 ft.;

a = the distance from the top of the dam to the level of highest water surface = 10 ft.;

a' = the distance from the top of the dam to the level of the spillway crest = 20 ft.;

P_i = ice pressure = 40,000 lb. per lin. ft. of dam, assumed to act at the level of the spillway crest, or level 20.*

As the foundation is assumed to be rock, the allowed inclination of the down-stream face of the dam may be taken from Eq. (28), or, maximum allowed $\tan \phi' = \frac{4}{3}f = \frac{4\times 0.75}{3} = 1.0$. Therefore, maximum allowed angle $\phi' = 45°$.

* If a rise of water surface above the spillway crest can be caused only by a freshet in the stream, it is reasonable to assume that, during the period of high water, the ice in the pond will be incapable of exerting pressure on the dam, owing to the larger area of water surface.

It is seen that there are two conditions of loading, for full reservoir, for both of which the designing rules must be fulfilled.

First Condition.—Low water and ice pressure.
Second Condition.—High water and no ice pressure.

It is necessary, therefore, to determine, at every stage of the design, by inspection or trial, which of these loadings will impose the more severe conditions. It will be found that, for this example, the first condition of loading will govern from the top down to Level 126.0 below which level the second condition must be used. It will be sufficient for the purpose of this example to consider in the explanation only that loading which, for any particular joint, is the determining condition.

The width of top and superelevation having been adopted, the first consideration is to ascertain whether there is sufficient masonry above the level of the application of ice pressure to fulfill the conditions of Rule 2. Referring to Art 32, the equation of investigation for Rule 2 is

$$\tan\theta = \frac{\Sigma(P)}{\Sigma(W)}.$$

Above Level 20:

$$\Sigma(P) = \text{Ice pressure} = 40{,}000 \text{ lb.};$$
$$\Sigma(W) = \text{weight of masonry} = 24\times20\times145 = 69{,}600 \text{ lb.}$$
$$\tan\theta = \frac{40{,}000}{69{,}600} = 0.574,$$

which is seen to be well within the allowed value of 0.75.

The next consideration is to determine the level of the bottom of Zone II, Fig. 14, or at what level the down-stream face must begin to batter. This evidently comes under the third case given in Art. 31, and Eq. (21) will apply.

$$(cw_2L^2+6P_i-w_1L^2)h+w_2h^3=w_1L^2a,$$

$$(0.5\times62.5\times24\times24+6\times40{,}000-145\times24\times24)h+62.5h^3 = 145\times24\times24\times20.$$

This equation must be solved by trial. The final value will be found to be $h=9.27$, and the bottom of Zone II will be 9.27 ft. below the water surface, or at Level 20+9.27=29.27.

It is evident that the resultant, reservoir empty, intersects the center of the joint as long as both faces are vertical. With the resultant, reservoir empty, at the center of the joint and the resultant, reservoir full, at its down-stream extremity, all conditions of Rule 1 are met.

For Rule 2 we have, from Eq. (22)

$$\tan\theta=\frac{\Sigma(P)}{\Sigma(W)}.$$

$\Sigma(P)=$

Ice pressure	40,000 lb.
Water pressure $\dfrac{9.27\times 9.27\times 62.5}{2}=$	2,685 lb.
	42,685 lb.

$\Sigma(W)=$

Masonry $24\times 29.27\times 145$	$=101,800$ lb.
Uplift $\dfrac{9.27\times 62.5\times 24\times 0.5}{2}$	$=-3,475$ lb.
	98,325 lb.

$$\text{Tan}\ \theta=\frac{42,685}{98,325}=0.434,$$

which is well within the allowed value of 0.75.

The requirement of limiting compressive stresses (Rule 3), is never a governing condition for solid dams less than 100 ft. high, on good rock foundations.

As the down-stream face is vertical, Rule 4 is not a governing condition.

Therefore, it is proved that the dam above Level 29.27 is stable, and we are now ready to proceed to the design of lower joints. The height of the dam should now be divided into a number of parts by imaginary horizontal joints, each joint being a distance from the one next above not greater than about 15 per cent of the distance of the former below the top of the dam. Such joints are indicated in Fig. 21, at Levels 29.27, 34.00, 39.0, etc. It now remains to determine the length and location of such joints.

The joint at Level 34.0 is evidently in Zone III, and Case 2, of Art. 31, applies. Choosing the point of reference on the vertical

up-stream face of the dam; y, in Eq. (19), is zero, and there results,

$$\frac{2l}{3}=\frac{\Sigma(Wx)_F+\Sigma(Px)_F}{\Sigma(W)_F}. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (19a)$$

In order to solve this equation, we must assume, tentatively, from a general knowledge of the trend of the down-stream faces of dams of this type, a length of joint with which to derive factors for substitution in the second term. Assume, for the first trial, a length of 26.0 ft., as indicated in Fig. 18. A convenient arrangement of the necessary calculations is indicated in Table VII.

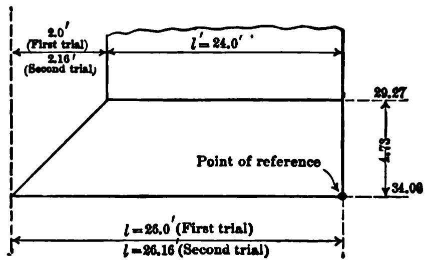


FIG. 18.

Making the necessary substitutions, from Table VII, in Eq. (19a), there results,

$$\frac{2l}{3}=\frac{1,975,400}{113,256}=17.44;$$

$$l=26.160.$$

It is seen that our first assumption of $l=26.0$ ft. was a little too small. Using the value, $l=26.16$, as a new tentative value of length of joint, as indicated in Fig. 18, we now construct Table VIII.

Making the proper substitutions in Eq. (19a), there results,

$$\frac{2l}{3}=\frac{1,976,210}{113,279}=17.45;$$

$$l=26.17;$$

this value of l is seen to agree almost exactly with the value assumed, and will be called final. Forces and moments, for use in

TABLE VII

COMPUTATIONS FOR JOINT AT LEVEL 34.00.—FIRST TRIAL

Item.		Factors.		Force.	Lever.		Moment
Vertical Forces.	Masonry above level 29.27	From previous calculation:		101,800	12.00		1,221,600
	Masonry below level 29.27	24 ×4.73 ×145		16,460	12.00		197,600
		2 ×4.73 ×145 ÷2		686	24.67		16,920
			$\Sigma(W)_E$	118,946		$\Sigma(Wx)_E$	1,436,120
	Uplift pressure	14 ×62.5 ×26 ×0.5 ÷2		−5,690	8.67		−49,320
			$\Sigma(W)_F$	113,256		$\Sigma(Wx)_F$	1,386,800
Horizontal Forces.	Ice pressure	By assumption:		40,000	14.00		560,000
	Head-water pressure	14 ×14 ×62.5 ÷2		6,120	4.67		28,600
			$\Sigma(P)_F$	46,120		$\Sigma(Px)_F$	588,600
					$\Sigma(Wx)_F+\Sigma(Px)_F$		1,975,400

TABLE VIII

COMPUTATIONS FOR JOINT AT LEVEL 34.00.—SECOND TRIAL

Item.		Factors.		Force.	Lever.		Moment.
Vertical Forces.	Masonry above level 29.27	From previous calculation		101,800	12.00		1,221,600
	Masonry below level 29.27	24 ×4.73 ×145		16,460	12.00		197,600
		2.16 ×4.73 ×145 ÷2		744	24.72		18,410
			$\Sigma(W)_E$	119,004		$\Sigma(Wx)_E$	1,437,510
	Uplift pressure	14 ×62.5 ×26.16 ×0.5 ÷2		−5,725	8.72		−49,900
			$\Sigma(W)_F$	113,279		$\Sigma(Wx)_F$	1,387,610
Horizontal Forces.	Ice pressure	By assumption		40,000	14.00		560,000
	Head-water pressure	14 ×14 ×62.5 ÷2		6,120	4.67		28,600
			$\Sigma(P)_F$	46,120		$\Sigma(Px)_F$	588,600
					$\Sigma(Wx)_F+\Sigma(Px)_F$		1,976,210

subsequent calculations should, of course, be taken from Table VIII. The two sets of calculations afford a valuable check on one another.

To determine the location of the resultant for any predesigned section, we may use Eq. (18). Placing the subscript, $_E$, to represent an investigation for empty reservoir, we have:

$$z=\frac{\Sigma(Wx)_E+\Sigma(Px)_E}{\Sigma(W)_E}.$$

From Table VIII we get

$$z=\frac{1,437,510+0\ ^*}{119,004}=12.07.$$

The distance from the point of reference to the up-stream extremity of the middle third being $26.17\div3=8.72$ ft., it is seen that the resultant, reservoir empty, lies well within the middle third. Since the resultant, reservoir full, has been made to intersect at the down-stream extremity of the middle third, all conditions of Rule 1 have been observed.

To test for Rule 2, we have,

$$\tan\theta=\frac{\Sigma(P)_F}{\Sigma(W)_F}=\frac{46,120}{113,279}=0.407,$$

which is well within the allowed value of 0.75.

As explained heretofore, the requirement of limiting compressive stresses (Rule 3) is not a governing condition for this height of dam.

For Rule 4, we have,

$$\tan\phi=\frac{2.17}{4.73}=0.459,$$

which is well within the allowed value of 1.0.

Thus we have designed and tested the dam above Level 34.0.

The design is continued in the same manner to the bottom of Zone III. The results of all calculations should be tabulated, as

* In every case, that condition of loading which will give the severest requirements for the up-stream portion of the joint should be taken for "empty reservoir." Usually, the assumption that the reservoir is empty will result in the greatest pressures at the heel, and the nearest location of the resultant to the up-stream extremity of the middle third.

TABLE IX

COMPUTATIONS FOR JOINT AT LEVEL 102.0.—FIRST TRIAL

Item.		Factors.		Force.	Lever.		Moment.
Vertical Forces.	Masonry above level 87.0	From previous calculation		409,000	27.35		11,190,000
	Masonry below level 87.0	0.3×15×145÷2		326	9.90		3,230
		50.56×15×145		110,200	35.32		3,892,000
		8.55×15×145÷2		9,300	63.50		590,500
			$\Sigma(W)_E$	528,826		$\Sigma(Wx)_E$	15,675,730
	Vertical component of head-water pressure	0.3×67×62.5		1,256	9.85		12,370
		0.3×15×62.5÷2		100	9.80		1,000
				530,182			15,689,100
	Uplift pressure	82×59.5×62.5×0.5÷2		−76,200	29.53		−2,250,000
			$\Sigma(W)_F$	453,982		$\Sigma(Wx)_F$	13,439,100
Horizontal Forces.	Ice pressure	By assumption		40,000	82.00		3,280,000
	Horizontal component of head-water pressure	82×82×62.5÷2		210,100	27.33		5,741,000
			$\Sigma(P)_F$	250,100		$\Sigma(Px)_F$	9,021,000
					$\Sigma(Wx)_F+\Sigma(Px)_F$		22,460,100

in Table XIII, and the location of the resultants, tan θ, and other characteristics plotted for both conditions of loading, as in Fig. 21, to an extent sufficient at least to foresee when it will be necessary to change from one zone to the other, or from one condition of loading to another, in the determination of the shape of the section.

Ordinarily, it is not necessary to calculate the compressive stresses occurring in the upper part of the dam. This has been done for Fig. 21, however, for the reader's information as to the variation of such stresses.

It is seen, by referring to Fig. 21, that, when Level 87.0 is reached, if the up-stream face is continued vertical, the resultant, reservoir empty, will intersect outside the middle third at the next joint to be designed. Level 87.0, therefore, marks the location of the last joint in Zone III, and the joint at Level 102.0 must be designed in accordance with the principle governing Zone IV. The first case of " Equations of Determination " of Art. 31 will apply. It will now be found convenient to locate the point of reference well outside the base, say 10 ft., as in Fig. 21. Eqs. (19) and (20) will apply to this case.

$$y+\frac{2l}{3}=\frac{\Sigma(Wx)_F+\Sigma(Px)_F}{\Sigma(W)_F}, \quad . \quad . \quad . \quad . \quad . \quad (19)$$

$$y+\frac{l}{3}=\frac{\Sigma(Wx)_E+\Sigma(Px)_E}{\Sigma(W)_E}. \quad . \quad . \quad . \quad . \quad . \quad (20)$$

It is now necessary to adopt tentative values of y and l (Fig. 13), for the derivation of the factors for substitution in the second term of each of these equations. From the general trend of the down-stream face of the dam, and a knowledge that the up-stream face must begin to batter slightly, values of $y=9.7$ and $l=59.5$ will be adopted, as indicated in Fig. 19.

Making the proper substitutions from Table IX in Eqs. (19) and (20), there results,

$$y+\frac{2l}{3}=\frac{22{,}460{,}100}{453{,}982}=49.47,$$

$$y+\frac{l}{3}=\frac{15{,}675{,}730+0}{528{,}826}=29.67,$$

Solving these two equations, there results:

$$l=59.40,$$

$$y=9.87.$$

These values of l and y are fairly close to the assumed values, namely $l=59.5$ and $y=9.7$, but closer values, as well as a check on the foregoing calculations, will be obtained by adopting them for new tentative values of l and y for a second set of calculations.

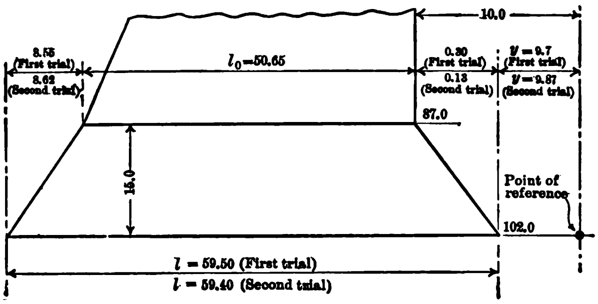


FIG. 19.

Substituting valves from Table X in Eqs. (19) and (20), there results,

$$y+\frac{2l}{3}=\frac{22,452,670}{453,553}=49.50,$$

$$y+\frac{l}{3}=\frac{15,679,410+0}{528,721}=29.68,$$

Combining these two equations, and solving for l and y, there results:

$$l=59.46,$$

$$y=9.86.$$

These values of l and y agree very closely with the tentative values, namely, $l=59.4$ and $y=9.87$, and will be called final. Thus, the joint has been designed in conformity with Rule 1.

To test for Rule 2, we have, for the first condition of loading,

$$\tan\theta=\frac{\Sigma(P)_F}{\Sigma(W)_F}=\frac{250,100}{453,553}=0.552.$$

Tan θ for the second condition of loading, determined in the same way, will be found to give the greater value, as indicated by Fig. 21. Tan θ, however, for both conditions of loading is seen to be well within the allowed value of 0.75.

TABLE X

COMPUTATIONS FOR JOINT AT LEVEL 102.0.—SECOND TRIAL

	Item.	Factors.		Force.	Lever.		Moment.
Vertical Forces.	Masonry above level 87.0	From previous calculation:		409,000	27.35		11,190,000
	Masonry below level 87.0	0.13×15×145÷2		141	9.96		1,410
		50.65×15×145		110,200	35.32		3,892,000
		8.62×15×145÷2		9,380	63.52		596,000
			$\Sigma(W)_E$	528,721		$\Sigma(Wx)_E$	15,679,410
	Vertical component of head-water pressure	0.13×67×62.5		871	9.94		8,660
		0.13×15×62.5÷2		61	9.92		600
				529,653			15,688,670
	Uplift pressure	82×59.4×62.5×0.5÷2		−76,100	29.67		−2,257,000
			$\Sigma(W)_F$	453,553		$\Sigma(Wx)_F$	13,431,670
Horizontal Forces.	Ice pressure	By assumption		40,000	82.00		3,280,000
	Horizontal component of head-water pressure	82×82×62.5÷2		210,100	27.33		5,741,000
			$\Sigma(P)_F$	250,100		$\Sigma(Px)_F$	[illegible]1,000
					$\Sigma(Wx)_F+\Sigma(Px)_F$		22,452,670

It will be evident, from a little study, that the maximum compressive stresses, both inclined and vertical, will occur at the toe of the joint for full reservoir, and at the heel for empty reservoir.

For full reservoir we may substitute in Eq. (10a)* of Art. 33,

$$\Sigma(W)=\Sigma(W)_F=453{,}550 \text{ from Table X,}$$

$$l=59.46,$$

$$u=\frac{l}{3}=19.82,$$

$$p_u'=0, \text{ as there is no tail-water.}$$

The maximum vertical compressive stress for full reservoir is then found to be:

$$p_v'=\frac{2\times 453{,}550}{59.46}\left(2-\frac{3\times 19.82}{59.46}\right)+0=15{,}250.$$

For empty reservoir, Eq. (11a) * applies,

$$\Sigma(W)=\Sigma(W)_E=528{,}720,$$

$$u=\frac{2l}{3}=39.64,$$

$$p_u''=0,$$

as there is, for empty reservoir, no head-water pressure.

The maximum vertical compressive stress for empty reservoir is then found to be:

$$p_v''=\frac{2\times 528{,}720}{59.46}\left(\frac{3\times 39.64}{59.46}-1\right)+0=17{,}770.$$

For the maximum inclined compressive stresses in the dam, Eqs. (23a) and (23b) apply. Eqs. (24a) and (24b) have no prac-

* As this is a special case, with the resultant, reservoir full and empty, at the exact extremity of the middle third, special Eqs. (12a) and (13a) of Art. 22, would be more simple of application. In order to reduce the number of equations to be dealt with, these equations have not been used in the examples, but may be readily applied by the reader, simply by omitting the parts of Eqs. (10a) and (11a) in parenthesis, which, for this special case, are equal to unity.

tical use unless the strength of the foundation is less than that of the masonry.

At the toe of the dam, $\tan\phi' = \frac{8.67}{15} = 0.577$; $\sec^2\phi' = 1.330$.

At the heel of the dam, $\tan\phi'' = \frac{0.12}{15} = 0.0080$; $\sec^2\phi'' = 1.00$.

For full reservoir, $\tan\theta = 0.552$; $\sec^2\theta = 1.305$.

For empty reservoir, $\tan\theta = 0$; $\sec^2\theta = 1.00$.

p_n is zero at the toe for full reservoir and at the heel for empty reservoir.

Using these values in Eq. (23*a*), the maximum inclined compressive stress for full reservoir is found to be:

$$P_i' = (15{,}250 \times 1.330 - 0) \text{ or } 0, \text{ or } (15.250 \times 1.305),$$
$$P_i' = 20{,}300, \text{ the greatest of these values.}$$

In the same way, from Eq. (23*b*), the maximum inclined compressive stress for empty reservoir is found to be:

$$P_i'' = (17{,}770 \times 1 - 0), \text{ or } 0, \text{ or } (17{,}770 \times 1),$$
$$P_i'' = 17{,}770, \text{ the greatest of these values.}$$

All vertical and inclined pressures at this joint are seen to be well within the allowed values, and, therefore, Rule 3 is followed.

To test for Rule 4, we have only to observe that the value of $\tan\phi'$, as just derived, is well within the allowed value of 1.0.

Thus we have designed and tested the section above level 102.0.

The design is continued in the same manner to the bottom of Zone IV. After the joint at Level 120.0 has been reached, it will be seen, by reference to Fig. 21, that the lines of resultants for the two conditions of loading will cross before the next joint is reached. It will be obvious, therefore, that the second condition of loading will govern from here on; namely, water surface at Level 10 and no ice pressure.

When the joint at Level 140.00 has been designed, the diagram on Fig. 21 will indicate that, unless proper precaution is taken, the vertical pressures at the toe for full reservoir will exceed the allowed limit of 18,000. Level 140.0, therefore, marks the elevation of the last joint in Zone IV, and the joint at Level 160.0 must be designed

in accordance with the principles governing Zone V, namely, Rule 3 for full reservoir and Rule 1 for empty reservoir. Eqs. (25) and (20), of Art. 33, apply to this case.

$$\frac{p_u' - p_v'}{6}l^2 - \frac{\Sigma(W)_F}{3}l = \Sigma(W)_F y - \{\Sigma(Wx)_F + \Sigma(Px)_F\}. \quad . \quad (25)$$

$$y + \frac{l}{3} = \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E}. \quad . \quad . \quad . \quad . \quad . \quad (20)$$

As before, it is necessary to adopt tentative values of l and y. These will be assumed at 107.0 and 6.0, respectively.

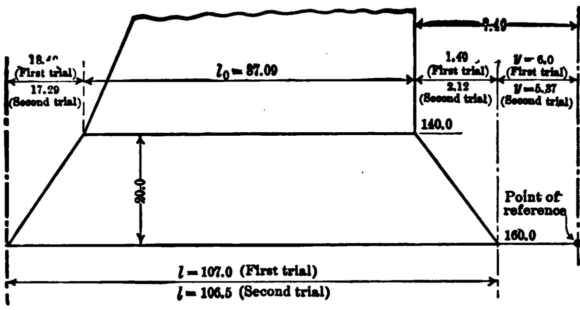


FIG. 20.

Making the proper substitutions from Table XI in Eq. (25), there results,

$$\frac{0 - 18{,}000}{6}l^2 - \frac{991{,}900}{3}l = 991{,}900y - 74{,}513{,}440,$$

which reduces to

$$l^2 + 110.21l = -330.63y + 24{,}838. \quad . \quad . \quad (25a)$$

Making corresponding substitutions in Eq. (20), there results:

$$y + \frac{l}{3} = \frac{49{,}580{,}000 + 0}{1{,}212{,}780} = 40.87. \quad . \quad . \quad . \quad . \quad (20a)$$

Combining Eqs. (20a) and (25a), and solving for l and y, there results,

$$l = 106.5,$$
$$y = 5.37.$$

TABLE XI

COMPUTATIONS FOR JOINT AT LEVEL 160.0.—FIRST TRIAL

Item.		Factors.		Force.	Lever.		Moment.
Vertical Forces.	Masonry above level 140	From previous calculation		931,200			33,985,000
	Masonry below level 140	1.49 × 20 × 145 ÷ 2		2,160	6.99		15,100
		87.09 × 20 × 145		252,700	51.03		12,890,000
		18.42 × 20 × 145 ÷ 2		26,720	100.72		2,690,000
			$\Sigma(W)_E$	1,212,780		$\Sigma(Wx)_E$	49,580,000
	Vertical component of head-water pressure	Above level 140, from previous calculation		16,780			145,740
		1.49 × 130 × 62.5		12,110	6.74		81,650
		1.49 × 20 × 62.5 ÷ 2		930	6.50		6,050
				1,242,700			49,813,440
	Uplift pressure	150 × 107 × 62.5 × 0.5 ÷ 2		−250,800	41.67		−10,450,000
			$\Sigma(W)_F$	991,900		$\Sigma(Wx)_F$	39,363,440
Horizontal Forces.	Horizontal component of head-water pressure	150 × 150 × 62.5 ÷ 2	$\Sigma(P)_F$	703,500	50.00	$\Sigma(Px)_F$	35,150,000
					$\Sigma(Wx)_F + \Sigma(Px)_F$		74,513,440

TABLE XII

COMPUTATIONS FOR JOINT AT LEVEL 160.0.—SECOND TRIAL

Item.		Factors.		Force.	Lever.		Moment.
Vertical Forces.	Masonry above level 140	From previous calculation		931,200			33,985,000
	Masonry below level 140	2.12×20×145÷2		3,070	6.78		20,830
		87.09×20×145		252,600	51.03		12,890,000
		17.29×20×145÷2		25,080	100.34		2,515,000
			$\Sigma(W)_E$	1,211,950		$\Sigma(Wx)_E$	49,410,830
	Vertical component of head-water pressure	Above level 140, from previous calculation		16,780			145,740
		2.12×130×62.5		17,220	6.43		110,800
		2.12×20×62.5÷2		1,320	6.08		8,060
				1,247,270			49,675,430
	Uplift pressure	150×62.5×106.5×0.5÷2		−249,800	40.87		−10,200,000
			$\Sigma(W)_F$	997,470		$\Sigma(Wx)_F$	39,475,430
Horizontal Forces.	Horizontal component of head-water pressure	150×150×62.5÷2	$\Sigma(P)_F$	703,000	50.00	$\Sigma(Px)_F$	35,150,000
					$\Sigma(Wx)_F+\Sigma(Px)_F$		74,625,430

These values of l and y are somewhat different from the values, $l=107.0$ and $y=6.0$ adopted tentatively. Closer values, as well as a check on the calculations, will be obtained by using them for new tentative values for a second set of calculations.

Making the proper substitutions from Table XII in Eq. (25), there results,

$$\frac{0-18,000}{6}l^2-\frac{997,470}{3}l=997,470y-74,625,430,$$

which reduces to

$$l^2+110.83l=-332.49y+24,875. \quad . \quad . \quad (25a')$$

Making corresponding substitutions in Eq. (20), there results,

$$y+\frac{l}{3}=\frac{49,410,830}{1,211,950}=40.80. \quad . \quad . \quad . \quad . \quad (20a')$$

Combining Eqs. (20a′) and (25a′), and solving for l and y, there results,

$$l=106.4,$$

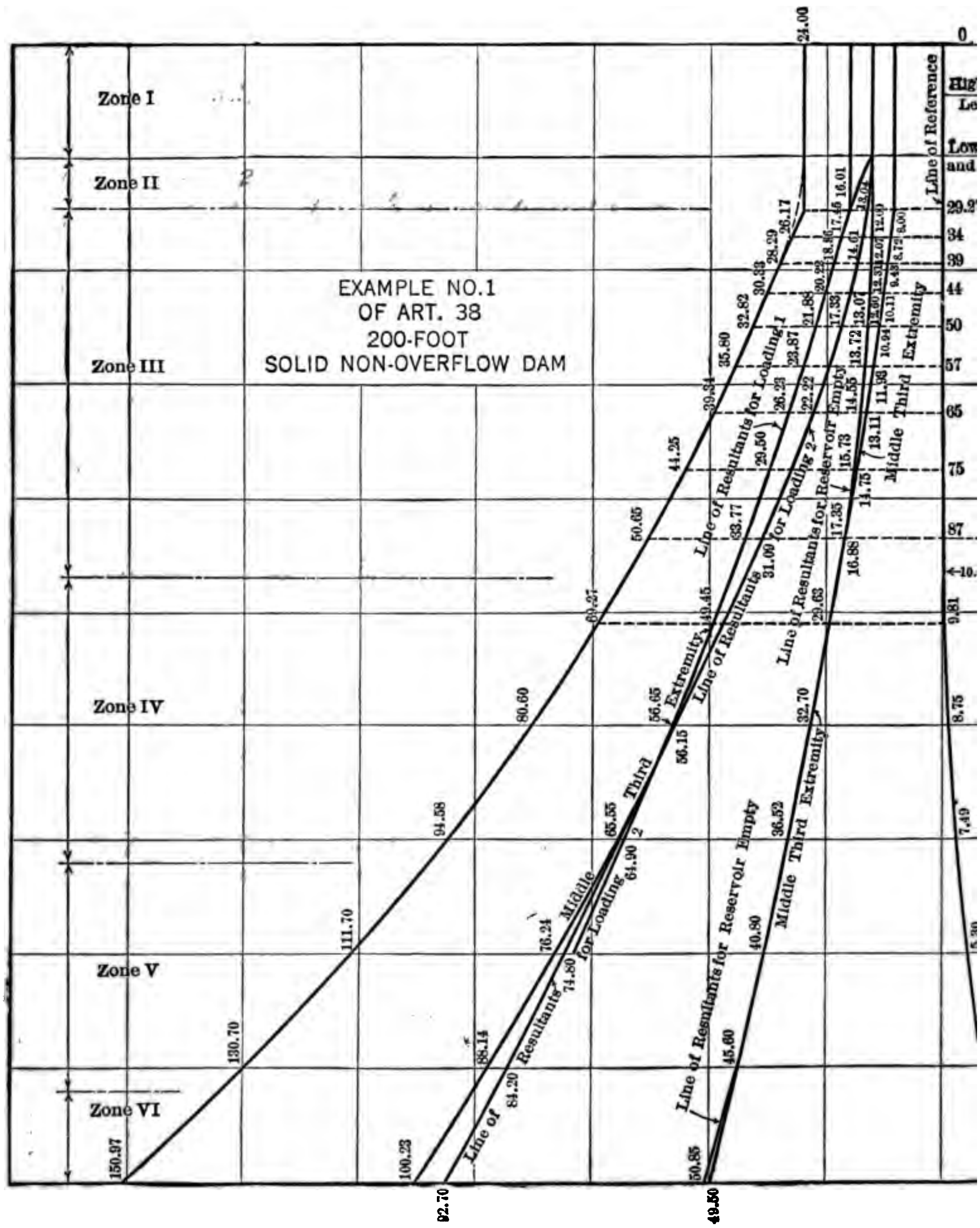
$$y=5.30;$$

which is sufficiently close to the tentative values of $l=106.5$ and $y=5.37$ and will be considered as final.

The necessary investigations to determine whether or not the other designing rules are complied with are, in every respect, similar to those described for the upper joints, and need not be repeated. The results are indicated in Fig. 21.

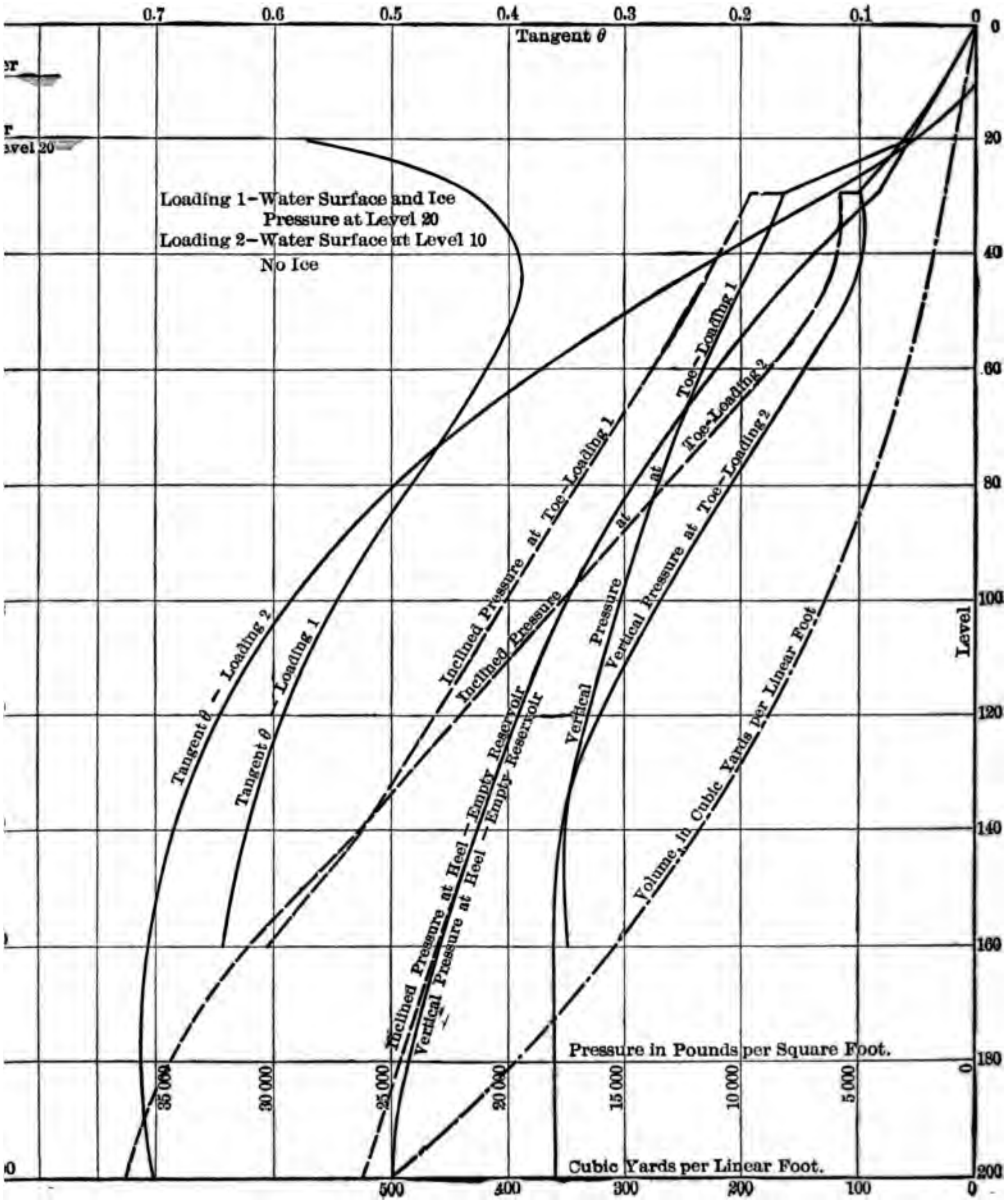
The joint at Level 180 is determined in the same manner, and, after the necessary calculations have been completed and plotted in Fig. 21, it will be noticed that, unless proper precautions are taken, the maximum allowed vertical pressures at the heel, for empty reservoir, will be exceeded. The joint at Level 200, therefore, will lie within Zone VI, and must be designed by the application of Eqs. (26) and (25), of Art. 33, in the same manner as Eqs. (25) and (20) were applied in the determination of the length and location of the joint at Level 160.

The angles in the faces of the designed section may be smoothed up, if desired, as indicated by the broken line for the angle at Level 29.27. In low dams the down-stream face is usually made straight, as indicated in Fig. 35, in order to save the expense of curved forms for the concrete.



EXAMPLE NO.1
OF ART. 38
200-FOOT
SOLID NON-OVERFLOW DAM
Zone I
Zone II
Zone III
Zone IV
Zone V
Zone VI
Line of Reference
Line of Resultants for Loading 1
Line of Resultants for Loading 2
Line of Resultants for Reservoir Empty
Middle Third Extremity
24.00
0
150.97
130.70
111.70
94.58
80.60
69.37
50.65
44.25
39.34
35.80
32.82
30.33
28.29
26.17
100.23
92.70
49.50
50.85

F



21.

To face page 95

It will be noticed that the design has not been influenced by the maximum allowed values of tan θ, tan ϕ', or maximum inclined pressures. Although this is usually the case for solid dams not exceeding 200 ft. in height, it will be well to indicate here the proper method of procedure, if, at any stage of the design, it is found that such values are determining conditions.

Referring to Fig. 21, it is seen that tan θ reaches a maximum value of 0.712 at Level 180.0. If it is considered that this value is too high, it will be necessary to provide an additional vertical, downward force, namely; an increased $\Sigma(W)$, as it will be seen from Eq. (22) that tan θ decreases as $\Sigma(W)$ increases. This may be accomplished simply by adding more masonry, but, usually, it will be found best to increase the batter of the up-stream face, as by doing this, a larger vertical component of head-water pressure will be included.

A reduction in the value of tan ϕ' may be made in a number of ways. If the section is redesigned with a greater superelevation or width of top, the resulting value of tan ϕ' will be less; but, in very high dams, the change will be relatively small. The desired reduction may also be obtained by arbitrarily thickening the middle and upper part of the section and redesigning the lower part. Such increase in thickness can be obtained by increasing the batter of the up- or down-stream faces. It is probable that the best remedial expedient, for most cases, is to increase the batter of the up-stream face from about mid-height to the base, and to redesign the lower part of the dam with the new up-stream face as a fixed condition in the calculations.

The inclined pressures, for a dam of this type, will always be greater at the toe than at the heel, notwithstanding the fact that the opposite is true of the allowed vertical pressures. It is seen from Eq. (23a) and (24a), that the maximum inclined stress in the masonry at the toe of the dam is proportional to the vertical stress, p_v', and to sec ϕ' or sec θ, whichever is the greater; and that the maximum inclined stress in the foundation is proportional to p_v' and sec θ. It will be found that a reduction in the inclined stresses in the dam can be obtained to best advantage by reducing sec ϕ', if ϕ' is greater than θ, or by reducing p_v', if θ is greater than ϕ'. To reduce the inclined stress in the foundation, a reduction in p_v' will usually prove best.

TABLE XIII

RESULTS OF CALCULATIONS FOR EXAMPLE NO. 1 OF ART. 38

Level of Joint.	Vertical Forces.							
	Weight of Masonry. $\Sigma(W)_E$.	Vertical Component of Head-water Pressure.		Uplift. Pressure.		Summation of Vertical Forces, Reservoir Full, $\Sigma(W)_F$.		
	Reservoir Empty.	Loading 1.	Loading 2.	Loading 1.	Loading 2.	Loading 1.	Loading 2.
29.27	101,800	....	....	3,475	7,230	98,320	94,570
34	119,000	....	....	5,725	......	113,300	
39	138,700	....	....	8,400	12,820	130,300	125,900
44	160,000	....	....	11,380	....	148,600	
50	187,500	....	....	15,400	20,500	172,100	167,000
57	222,300	....	....	20,700	....	201,600	
65	265,900	....	....	27,700	33,830	238,200	232,100
75	326,500	....	....	38,050	....	288,500	
87	409,000	....	....	53,050	60,950	356,000	348,100
102	528,700	932	....	76,100	....	454,000	
120	700,700	8,096	7,700	112,800	123,600	596,000	584,800
140	931,300	16,770	16,780	163,200	176,800	784,800	771,300
160	1,212,000	34,540	35,320	232,500	249,800	1,014,000	997,500
180	1,551,000	....	56,320	....	338,300	....	1,296,000
200	1,957,000	....	103,300	....	453,000	....	1,607,000

Level of Joint.	Horizontal Forces.				
	Horizontal Ice Pressure.	Horizontal Component of Head-water Pressure.		Summation of Horizontal Forces, Reservoir Full, $\Sigma(P)_F$.	
	Loading 1.	Loading 1.	Loading 2.	Loading 1.	Loading 2.
29.27	40,000	2,685	11,600	42,685	11,600
34	40,000	6,120	....	46,120	
39	40,000	11,280	26,280	51,280	26,280
44	40,000	18,000	....	58,000	
50	40,000	28,120	50,000	68,120	50,000
57	40,000	42,750	....	82,750	
65	40,000	63,300	94,550	103,300	94,550
75	40,000	94,600	....	134,600	
87	40,000	140,200	185,300	180,200	185,300
102	40,000	210,100	....	250,100	
120	40,000	312,500	378,000	352,500	378,000
140	40,000	450,000	528,100	490,000	528,100
160	40,000	612,000	703,000	652,000	703,000
180	40,000	....	903,000	....	903,000
200	40,000	....	1,128,000	....	1,128,000

TABLE XIII—*Continued*

Level of Joint.	MOMENTS OF VERTICAL FORCES. Moment of Weight of Masonry $\Sigma(Wx)_E$.	Moment of Vertical Component of Head-water Pressure.		Moment of Uplift Pressure.	
	Reservoir Empty.	Loading 1.	Loading 2.	Loading 1.	Loading 2.
29.27	1,222,000	....	....	27,800	57,850
34	1,438,000	....	....	49,900	
39	1,706,000	....	....	79,200	120,900
44	2,018,000	....	....	115,000	
50	2,451,000	....	....	168,400	224,300
57	3,050,000	....	....	247,000	
65	3,869,000	....	....	363,000	443,500
75	5,137,000	....	....	561,000	
87	7,098,000	....	....	894,000	1,029,000
102	15,680,000	9,260	....	2,257,000	
120	22,930,000	75,490	72,040	3,688,000	4,040,000
140	33,980,000	145,700	145,700	5,965,000	6,465,000
160	49,410,000	258,700	264,600	9,480,000	10,200,000
180	70,680,000	....	352,400	....	15,420,000
200	99,570,000	....	393,400	....	22,490,000

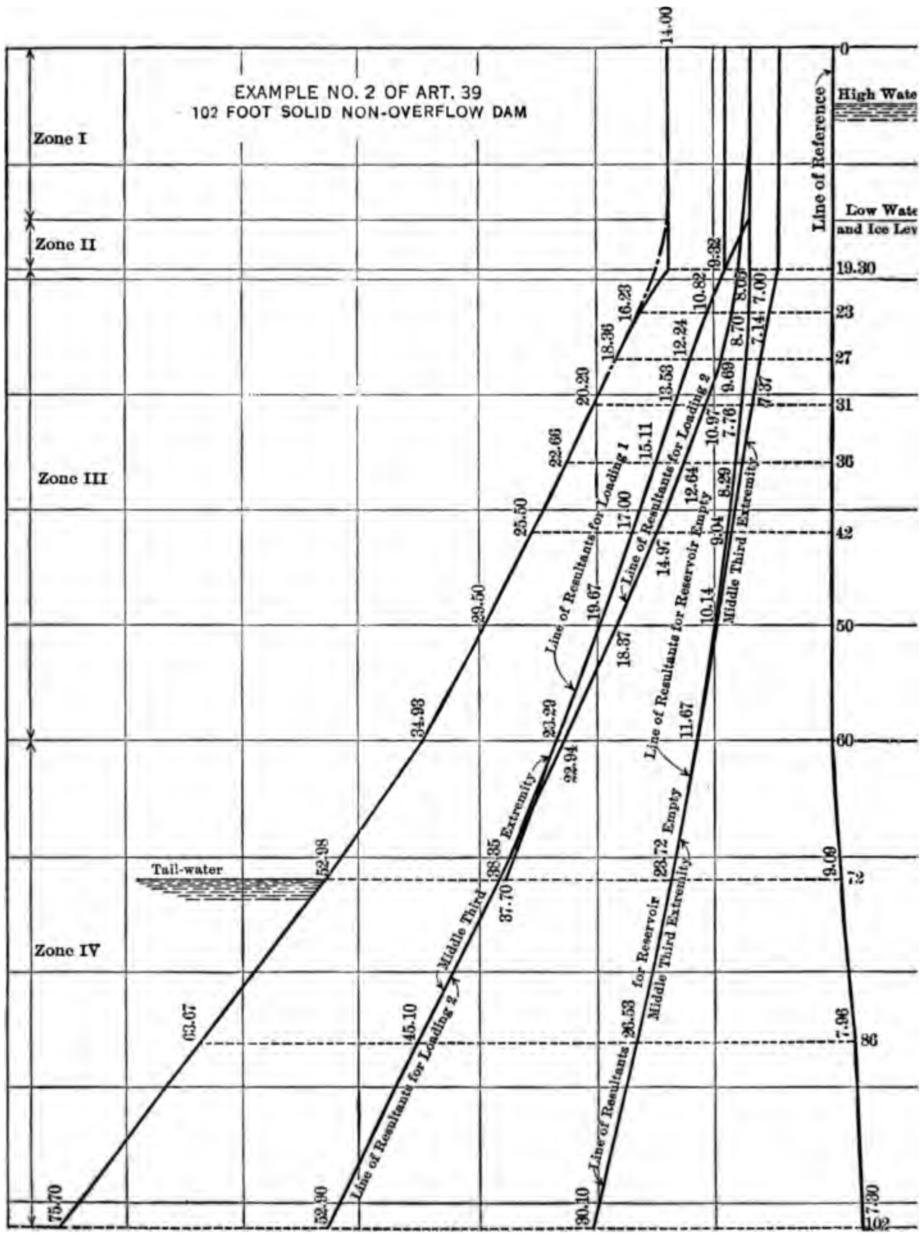
Level of Joint.	MOMENTS OF HORIZONTAL FORCES. Arm and Moment of Ice Pressure.		Arm and Moment of Horizontal Component of Head-water Pressure.				TOTAL MOMENTS. Summation of Moments of all Forces, Full Reservoir. $\Sigma(Wx)_F+\Sigma(Px)_F$	
	Loading 1.		Loading 1.		Loading 2.		Loading 1.	Loading 2.
29.27	9.27	370,800	3.09	8,300	6.42	74,450	1,573,000	1,238,000
34	14.00	560,000	4.67	28,600	....	....	1,976,000	
39	19.00	760,000	6.33	71,400	9.67	254,200	2,458,000	1,839,000
44	24.00	960,000	8.00	144,000	....	....	3,007,000	
50	30.00	1,200,000	10.00	281,200	13.33	666,500	3,764,000	2,894,000
57	37.00	1,480,000	12.33	527,500	....	....	4,810,000	
65	45.00	1,800,000	15.00	950,000	18.33	1,734,000	6,256,000	5,159,000
75	55.00	2,200,000	18.33	1,734,000	....	....	8,510,000	
87	67.00	2,680,000	22.33	3,133,000	25.67	4,755,000	12,020,000	10,820,000
102	82.00	3,280,000	27.33	5,741,000	....	....	22,450,000	
120	100.00	4,000,000	33.33	10,420,000	36.67	13,860,000	33,730,000	32,820,000
140	120.00	4,800,000	40.00	18,000,000	43.33	22,890,000	50,970,000	50,560,000
160	140.00	5,600,000	46.67	28,600,000	50.00	35,150,000	74,390,000	74,630,000
180	.....	....	....	....	56.67	51,220,000	....	106,800,000
200	.....	....	....	....	63.33	71,450,000	....	148,900,000

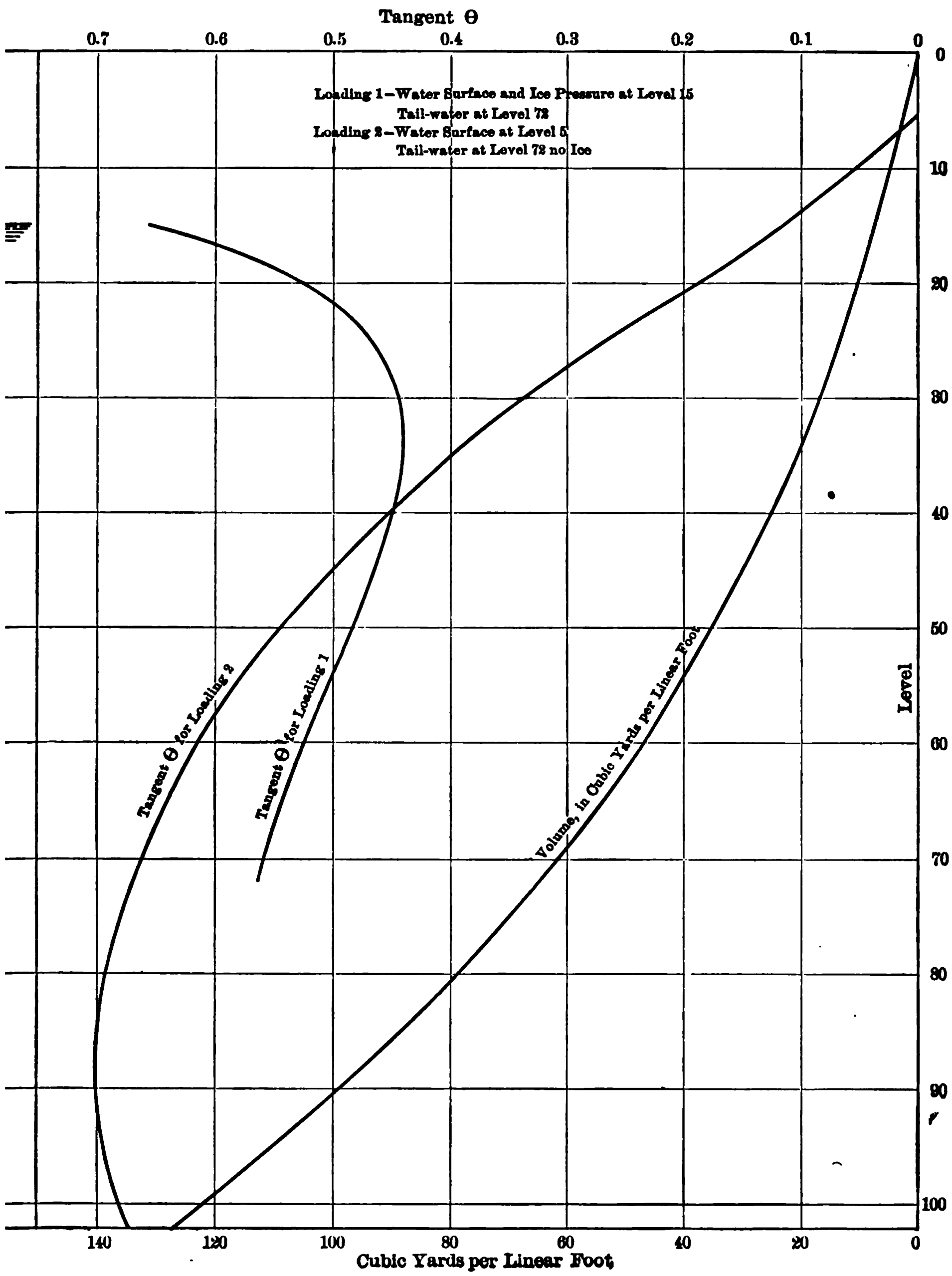
TABLE XIII—*Continued*

Level of Joint.	Location of Resultants. Distance from Point of Reference. Reservoir Empty.	Load-ing 1.	Load-ing 2.	Length of Base.	Distance of Heel from Point of Reference.	Tangent ϕ. ϕ'' Heel.	ϕ' Toe.*	Tangent θ. Load-ing 1.	Load-ing 2.
29.27	12.00	16.01	13.04	24.00	0	0	0	0.434	0.123
34	12.07	17.45	.....	26.17	0	0	0.459	0.407	
39	12.31	18.86	14.61	28.29	0	0	0.424	0.394	0.209
44	12.60	20.22	.....	30.33	0	0	0.408	0.390	
50	13.07	21.88	17.33	32.82	0	0	0.415	0.396	0.299
57	13.72	23.87	.....	35.80	0	0	0.426	0.410	
65	14.55	26.33	22.22	39.34	0	0	0.442	0.434	0.407
75	15.73	29.50	.....	44.25	0	0	0.491	0.467	
87	17.35	33.77	31.09	50.65	0	0	0.533	0.506	0.533
102	29.68	49.50	.....	59.46	9.87	0.009	0.575	0.552	
120	34.70	56.65	56.15	71.85	8.75	0.059	0.630	0.592	0.646
140	36.52	64.90	65.55	87.09	7.49	0.063	0.699	0.624	0.685
160	40.80	73.30	74.80	106.40	5.30	0.109	0.856	0.643	0.705
180	45.60	.....	84.20	127.70	3.00	0.115	0.950	......	0.712
200	50.85	.....	92.70	152.20	−1.23	0.212	1.013	......	0.702

Level of Joint.	Maximum Pressures, in Thousands of Pounds per Square Foot. At Heel. Reservoir Empty. Vertical.	Inclined.	At Toe. Loading 1. Vertical.	Inclined.	Loading 2. Vertical.	Inclined.
29.27	4.2	....	8.2	8.2	5.0	5.0
34						
39	6.6	....	9.2	10.9	4.9	5.8
44						
50	9.2	....	10.5	12.3	5.9	7.0
57						
65	12.0	....	12.1	14.5	8.2	9.8
75						
87	15.7	....	14.1	18.1	11.6	14.8
102	17.8	17.8	15.3	20.3		
120	19.5	19.5	16.6	23.1	15.9	22.2
140	21.4	21.4	17.6	26.3	17.7	26.4
160	22.8	23.1	17.5	30.3	18.0	31.2
180	24.3	24.6	....	....	18.1	34.4
200	25.0	26.2	....	....	18.0	36.4

* It will be noticed that the last value of tan ϕ' was slightly in excess of the allowed value of 1.0. The difference, however, was not considered important enough to warrant correction.


EXAMPLE NO. 2 OF ART. 39
102 FOOT SOLID NON-OVERFLOW DAM
Zone I
Zone II
Zone III
Zone IV
Line of Reference
High Wate
Low Wate
and Ice Lev
Tail-water
Line of Resultants for Loading 1
Line of Resultants for Loading 2
Line of Resultants for Reservoir Empty
Middle Third Extremity
Line of Resultants for Reservoir Empty
Middle Third Extremity
Line of Resultants for Loading 2
Middle Third Extremity
14.00
0
19.30
23
27
31
36
42
50
60
72
86
102
9.32
10.82
8.65
7.00
16.23
12.24
8.70
7.14
18.36
13.53
9.69
7.37
20.29
15.11
10.97
7.76
22.66
17.00
12.64
8.29
25.50
14.97
9.04
29.50
19.67
10.14
18.37
34.93
23.29
22.94
11.67
52.98
38.35
37.70
23.72
9.09
63.67
45.10
26.53
7.96
75.70
52.90
30.10
7.30



22. *To face page 99*

39. Example No. 2. 102-ft. Solid Non-Overflow Dam (Fig. 22). In this example the assumptions are the same as in Example No. 1, Art. 33, with the following exceptions:

H = maximum height of dam = 102 ft.;
L = width of top = 14 per cent of height = 14 ft.;
a = the distance from the top of the dam to the level of high-water surface = 5 ft.;
a' = the distance from the top of the dam to the level of the spillway crest = 15 ft.;
a'' = the distance from the top of the dam to tail-water surface = 72 ft.;
c = the area of joints and base subjected to uplift = 50 per cent. The uplift is assumed to vary uniformly from head-water pressure at the heel to tail-water pressure at the toe.

Except that the negative force and moment of tail-water is present, the method of design to be followed is exactly the same as that indicated for the upper part of Example No. 1, and will not be repeated.

The design is governed by low water, with ice pressure down to Level 65.0, and high water with no ice below that elevation. The results of computations are indicated in Table XIV, and are plotted in Fig. 22.

It will be noticed that the dam does not extend below Zone IV, the pressures at the base with resultants at the middle third extremities not exceeding the allowed values, as indicated by the calculations which follow.

It will be evident, from a little study, that the maximum compressive stresses, both inclined and vertical, will occur at the toe of the dam for full reservoir, and at the heel for empty reservoir.

For full reservoir we may substitute in Eq. (10a)*, of Art. 33, the following known values from Table XIV.

$$\Sigma(W) = \Sigma(W_F) = 395{,}700,$$

$$l = 68.4,$$

$$u = \frac{68.4}{3} = 22.8,$$

$$p_u' = 0.5 \times 30 \times 62.5 = 937.5.$$

* See footnote, p. 89.

TABLE XIV

RESULTS OF CALCULATIONS FOR EXAMPLE No. 2 OF ART. 39

Level of Joint.	VERTICAL FORCES.							
	Weight of Masonry $\Sigma(W)_E$.	Vertical Component of Head-water Pressure.		Vertical Component of Tail-water Pressure.	Uplift Pressure.		Summation of Vertical Forces, Reservoir Full $\Sigma(W)_F$.	
	Empty Reservoir.	Loading 1.	Loading 2.	Loading 2.	Loading 1.	Loading 2.	Loading 1.	Loading 2.
19.30	39,200	......	......	......	940	3,130	38,260	36,070
23	47,310	......	......	......	2,029	4,562	45,280	42,750
27	57,350	......	......	......	3,442	6,310	53,900	51,040
31	68,560	......	......	......	5,075	8,240	63,480	60,320
36	84,110	......	......	......	7,435	10,970	76,680	73,140
42	105,100	......	......	......	10,750	14,740	94,310	90,320
50	137,000	......	......	......	16,150	20,740	120,800	116,200
60	183,700	......	......	......	24,570	30,000	159,100	153,700
72	252,400	3,343	4,000	......	39,300	46,180	216,400	210,200
86	353,400	......	9,180	4,640	....	82,570	....	284,700
102	497,300	......	12,850	21,200	....	135,700	....	395,700

Level of Joint.	HORIZONTAL FORCES.					
	Horizontal Ice Pressure.	Horizontal Component of Head-water Pressure.		Horizontal Component of Tail-water Pressure.	Summation of Horizontal Forces, Reservoir Full, $\Sigma(P)_F$.	
	Loading 1.	Loading 1.	Loading 2.	Loading 2.	Loading 1.	Loading 2.
19.30	20,000	578	6,380	....	20,578	6,380
23	20,000	2,000	10,140	....	22,000	10,140
27	20,000	4,500	15,130	....	24,500	15,130
31	20,000	8,000	21,120	....	28,000	21,120
36	20,000	13,780	30,030	....	33,780	30,030
42	20,000	22,780	42,750	....	42,780	42,750
50	20,000	38,260	63,300	....	58,260	63,300
60	20,000	63,300	94,500	....	83,300	94,500
72	20,000	101,400	140,400	....	121,400	104,400
86	....	....	205,000	6,130	....	198,900
102	....	....	294,000	28,120	....	265,900

TABLE XIV—*Continued*

Level of Joint.	MOMENTS OF VERTICAL FORCES.					
	Moment of Weight of Masonry, $\Sigma(Wx)_E$.	Moment of Vertical Component of Head-water Pressure.		Moment of Vertical Component of Tail-water Pressure.	Moment of Uplift. Pressure.	
	Empty Reservoir.	Loading 1.	Loading 2.	Loading 2.	Loading 1.	Loading 2.
19.30	274,400	....	....	....	4,395	14,620
23	335,800	....	....	....	10,980	24,700
27	422,700	....	....	....	21,070	38,600
31	531,100	....	....	....	34,290	55,650
36	697,400	....	....	....	56,100	82,850
42	949,900	....	....	....	91,400	115,400
50	1,390,000	....	....	....	158,900	203,900
60	2,144,000	....	....	....	286,000	349,200
72	5,992,0 0	31,630	37,840	....	929,000	1,091,000
86	9,376,000	....	82,420	282,000	....	2,415,000
102	14,960,000	....	109,600	1,449,000	....	4,817,000

Level of Joint.	MOMENTS OF HORIZONTAL FORCES.							
	Arm and Moment of Ice Pressure.		Arm and Moment of Horizontal Component of Head-water Pressure.				Arm and Moment of Horizontal Component of Tail-water Pressure.	
	Loading 1.		Loading 1.		Loading 2.		Loading 2.	
19.30	4.30	86,000	1.43	826	4.77	30,400		
23	8.00	160,000	2.67	5,340	6.00	60,840		
27	12.00	240,000	4.00	18,000	7.33	110,900		
31	16.00	320,000	5.33	42,640	8.67	183,100		
36	21.00	420,000	7.00	96,460	10.33	310,300		
42	27.00	540,000	9.00	204,900	12.33	527,500		
50	35.00	700,000	11.67	446,500	15.00	949,500		
60	45.00	900,000	15.00	950,000	18.33	1,733,000		
72	57.00	1,140,000	19.00	1,926,000	22.33	3,135,000		
86	.....	....	.....	....	27.00	5,535,000	4.67	28,600
102	.....	....	.....	....	32.33	9,510,000	10.00	281,200

TABLE XIV—*Continued*

Level of Joint.	Total Moments. Summation of Moments of all Forces, Reservoir Full. $\Sigma(Wx)_F+\Sigma(Px)_F$.		Level of Joint.	Total Moments. Summation of Moments of all Forces, Reservoir Full. $\Sigma(Wx)_F+\Sigma(Px)_F$.	
	Loading 1.	Loading 2.		Loading 1.	Loading 2.
19.30	356,800	290,200	50	2,377,000	2,135,000
23	490,200	371,900	60	3,708,000	3,528,000
27	659,600	495,000	72	8,160,000	8,073,000
31	859,400	658,500	86	....	12,830,000
36	1,158,000	924,800	102	....	20,930,000
42	1,603,000	1,352,000			

Level of Joint.	Location of Resultants. Distance from Point of Reference.			Length of Base.	Distance of Heel from Point of Reference.	Tangent ϕ.		Tangent θ.	
	Empty Reservoir.	Loading 1.	Loading 2.			ϕ'' Heel.	ϕ' Toe.	Loading 1.	Loading 2.
19.30	7.00	9.32	8.05	14.00	0	0	0	0.538	0.176
23	7.14	10.82	8.70	16.23	0	0	0.603	0.486	0.237
27	7.37	12.24	9.69	18.36	0	0	0.532	0.455	0.296
31	7.76	13.53	10.92	20.29	0	0	0.482	0.442	0.351
36	8.29	15.11	12.64	22.66	0	0	0.475	0.441	0.410
42	9.04	17.00	14.97	25.50	0	0	0.473	0.457	0.473
50	10.14	19.67	18.37	29.50	0	0	0.500	0.482	0.544
60	11.67	23.29	22.94	34.93	0	0	0.543	0.523	0.614
72	23.72	37.70	38.35	43.89	9.09	0.076	0.671	0.561	0.667
86	26.53	.....	45.10	55.71	7.96	0.081	0.763	......	0.699
102	30.10	.....	52.90	68.40	7.30	0.041	0.752	......	0.672

Level of Joint.	Maximum Pressures, in Thousands of Pounds per Square Foot. At Heel. Empty Reservoir.		At Toe. Loading 2.	
	Vertical.	Inclined.	Vertical.	Inclined.
102	14.5	14.6	12.5	18.5

The maximum vertical compressive stresses for full reservoir is found to be:

$$p_v' = \frac{2\times 395{,}700}{68.4}\left(2-\frac{3\times 22.8}{68.4}\right)+937.5=12{,}500.$$

For empty reservoir, Eq. (11*a*) * applies,

$$\Sigma(W)=\Sigma(W)_E=497{,}300,$$

$$u=\frac{2\times 68.4}{3}=45.6,$$

$$p_u''=0.$$

The maximum vertical compressive stress for empty reservoir is then found to be:

$$p_v''=\frac{2\times 497{,}300}{68.4}\left(\frac{3\times 45.6}{68.4}-1\right)+0=14{,}550.$$

For the maximum inclined compressive stresses in the dam, Eqs. (23*a*) and (23*b*), of Art. 33, apply. Eqs. (24*a*) and (24*b*) have no practical use unless the strength of the foundation is less than that of the masonry.

At the toe of the dam,

$$\tan\phi'=\frac{12.03}{16}=0.752;\ \tan^2\phi'=0.566;\ \sec^2\phi'=1.568.$$

At the heel of the dam,

$$\tan\phi''=\frac{0.66}{16}=0.0412;\ \tan^2\phi''=0.0017;\ \sec^2\phi''=1.0016.$$

For full reservoir,

$\tan\theta=0.672;\quad \sec^2\theta=1.455.$

For empty reservoir,

$\tan\theta=0;\quad \sec^2\theta=1.00.$

Using these values in Eq. (23*a*), the maximum inclined compressive stress for full reservoir is found to be:

$$p_i'=(12{,}500\times 1.568-30\times 62.5\times 0.566)\ \text{or}\ 30\times 62.5\ \text{or}\ 12{,}500\times 1.455,$$

$$p_i'=18{,}540\ \text{or}\ 1{,}875\ \text{or}\ 18{,}200,$$

$p_i'=18{,}540$, the greatest of these values.

* See foot-note, p. 89.

In the same way, from Eq. (23*b*), the maximum inclined compressive stress for empty reservoir is found to be:

$$p_t'' = (14{,}550 - 0) \quad \text{or} \quad 0 \quad \text{or} \quad 14{,}550 \times 1.0016,$$
$$p_t'' = 14{,}550, \text{ the greatest of these values.}$$

40. Comparison of Non-overflow Dams. A comparison of high, solid, non-overflow dams is given in Fig. 23. With the exception of the Olive Bridge final section, the designs of these dams were all made in accordance with the same general theory, the differences in areas and shapes being affected solely by the assumptions, as indicated in Table XV.

The theoretical section of the Olive Bridge Dam was arbitrarily increased to its final section, on account of the importance of the structure.

TABLE XV

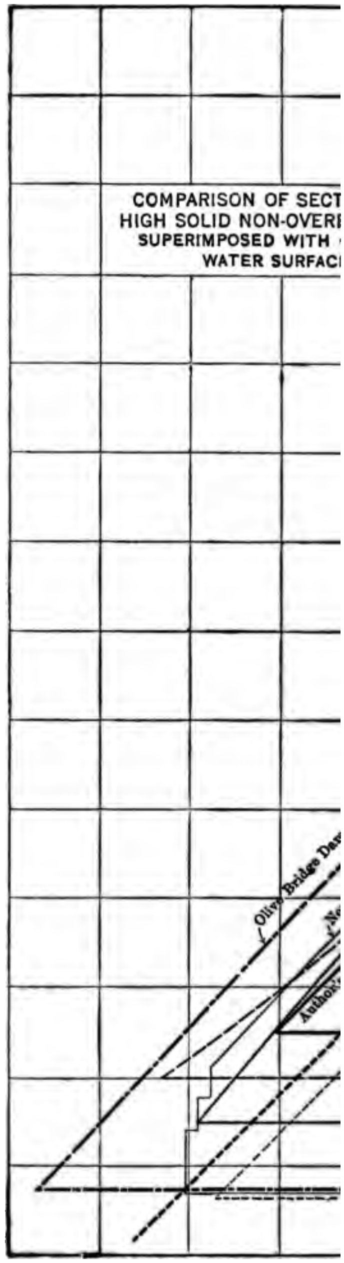
COMPARISON OF SOLID, NON-OVERFLOW DAMS, SHOWING ASSUMPTIONS USED IN DESIGN

(See Fig. 23)

Dam.	Unit Wt. of Masonry, in Lbs. per Cu.ft.	Percentage of Area of Base Subjected to Uplift.	Total Ice Pressure, in Lbs. per Lin.ft.	Maximum Allowed Vertical Pressures, in Lbs. per Sq.ft.	
				Toe.	Heel.
Olive Bridge, theoretical section	145.8	66⅔	47,000	40,000	40,000
Olive Bridge, final section	145.8	66⅔	47,000	12,200	23,000
New Croton	156.2	0	0	33,400	30,800
Elephant Butte	140.0	33⅓	0	22,000	28,000
Wegmann's Practical Profile No. 3	145.8	0	0	16,800	20,600
Morrison and Brodie's example of design	146.0	0	0	28,000	36,000
The author's Example No. 1	145.0	50	40,000	18,000	25,000

The maximum vertical pressures indicated for the Olive Bridge final section probably do not exist, as the maximum section is in a narrow gorge confined on both sides by good rock, between which the dam is wedged, without the possibility of movement.

In the upper part of the New Croton Dam the vertical pressures were limited to 16,400 and 20,600 lb. at the toe and heel, respectively, and, in the lower part, 33,400 and 30,800. This difference in allowed pressures, in the top and bottom of this dam, has been severely criticised.


COMPARISON OF SECT
HIGH SOLID NON-OVER
SUPERIMPOSED WITH
WATER SURFAC
Olive Bridge Dam

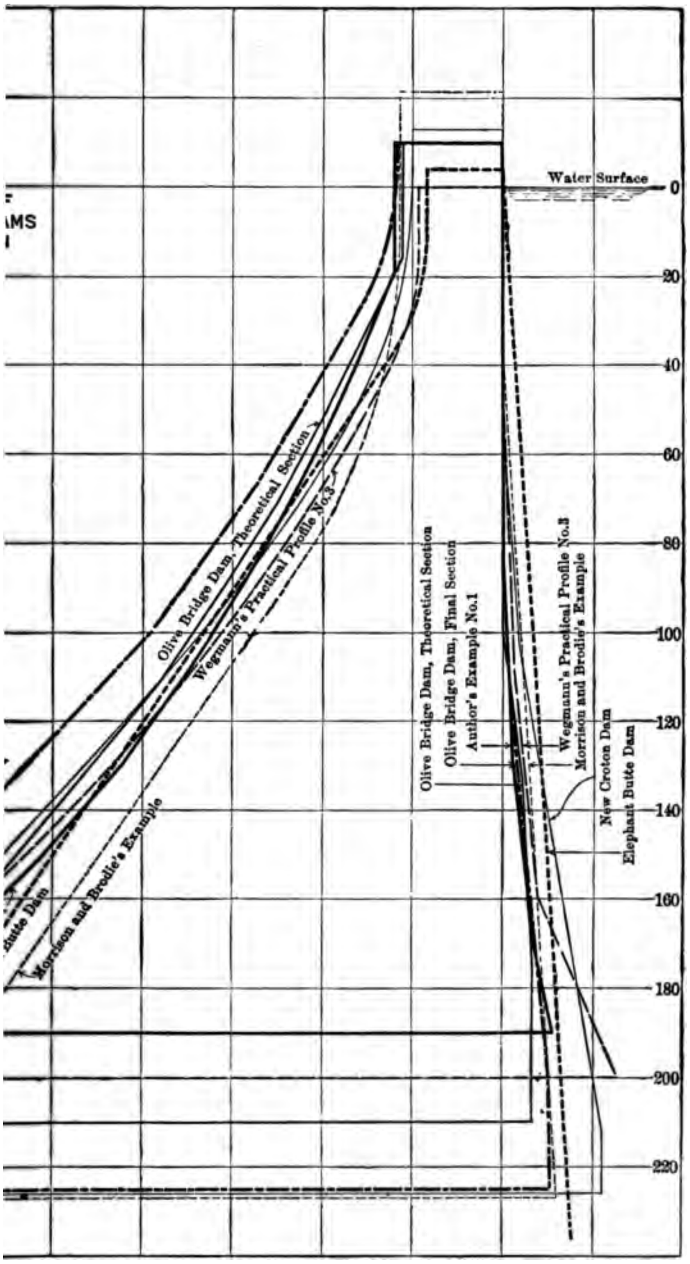


FIG. 23.

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CHAPTER VII

THE DESIGN OF SOLID SPILLWAY GRAVITY DAMS

41. General Considerations. The general method of design of solid spillway dams differs in no way from that previously described for solid non-overflow dams, except at the crest which, as mentioned in Art. 17, should be proportioned to fit the lower nappe of the sheet of water spilling over the dam during maximum flood.

The application of the general equations of design has been described in Example 1, Art. 38, and will not be repeated for the following examples.

42. The Shape of the Crest. It was pointed out in Art. 17 that, if the sheet of water spilling over the crest leaves the face of the masonry, there is danger of the existence of a considerable indeterminate overturning force due to the formation of a partial vacuum under the sheet. This, of course, will not occur if the continuous length of crest is unusually short and free access to the atmosphere is provided at the ends. At any rate, it is very desirable to avoid impact and vibration, and the best practice dictates that the crest and down-stream face should always completely fill the space under the lower nappe of the sheet corresponding to the maximum flood to be expected.

Experiments have been made to determine the shape of the sheet of water flowing over aerated sharp-crested weirs. The general form of the sheet is indicated in Fig. 24. If the area below the lower nappe is filled with masonry, the shape of the sheet and the discharge will not be changed appreciably. The resulting section will not only fit the sheet of water, but will provide the maximum possible discharge, as the water will pass the masonry crest with no disturbance or unnecessary contraction.

This shape of crest has become standard in modern designs and, with few exceptions, differs for individual cases only in the methods used in determining the shape of the sheet. For reference, therefore, it will be designated the "standard dam crest."

A comparison of the shape of the crests of several modern dams is made in Fig. 38.

Bazin's experiments for the shape of the sheet of water flowing over a sharp-crested weir have been translated by Messrs. Arthur Marichal and John C. Trautwine, Jr.* The experiments for weirs with vertical water faces and those with water faces inclined 45° apply directly to the determination of the shape of the crest of the ordinary types of solid and hollow dams, respectively. The curves indicated in Fig. 25 and 26 to about $x=+0.12$ for the upper nappe and $x=+0.65$ for the lower nappe are plotted directly from the experiments. The extension of the experimental data was made by the author in the following manner:

The average velocity in any normal section of the sheet of water was found by Bazin to lie very close to one-third of the distance from the lower nappe, as at point 4, Fig. 24. The curve, 2–3–4, was drawn through three scaled points of average or resultant velocities. The curve has the form of a jet spouting with an initial horizontal velocity, v_h. Its equation, therefore, is that of a parabola, and was derived as follows:

In the time, t, a particle on the curve will have fallen, from rest, a vertical distance from point 1, equal to

$$x'=\frac{t^2g}{2};$$

therefore,

$$t^2=\frac{2x'}{g}.$$

In this same time, t, the particle will have moved horizontally, from point 1, a distance of

$$y'=v_ht;$$

therefore,

$$t^2=\left(\frac{y'}{v_h}\right)^2.$$

Equating these two values of t^2, there results:

$$y'^2=\frac{2v_h^2}{g}x', \qquad \qquad (29a)$$

* Proceedings Engineers' Club of Philadelphia, Apr., 1893.

which is the equation of the curve, 1–2–3–4, referred to the origin at point 1.

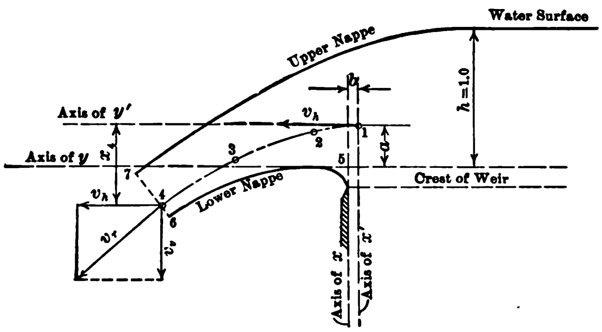


FIG. 24.

Referred to point 5, the equation is,

$$(y+b)^2=\frac{2v_h^2}{g}(x+a), \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

where x and y are co-ordinates referred to the origin, 5; and a, b and v_h, unknown constants.

Substituting in Eq. (29), measured values of y and x from each of the known points, 2, 3, and 4, of the experimental curve, there results three equations with three unknowns, a, b, and v_h. Values of a, b, and v_h, found in this manner for the two types of crests, and the result of the substitution of v_h in Eq. (29a), are given in Table XVIII (first four items). From Eq. (29a), the curve, 1–2–3–4, may be extended indefinitely. It now remains to determine the thickness of the jet at various points on the curve. The vertical velocity, v_v, at point 4 is

$$v_v=\sqrt{2gx'}.$$

The horizontal velocity, v_h, being known, the resultant velocity, v_r, is

$$v_r=\sqrt{v_h^2+v_v^2}=\sqrt{v_h^2+2gx'}.$$

Measuring the thickness, 6–7, of the sheet from the plot of the experiments, the discharge, q, per linear foot of crest may be calculated from the following equation:

$$q=Av_r=A\sqrt{v_h^2+2gx'}, \quad . \quad . \quad . \quad . \quad . \quad (30)$$

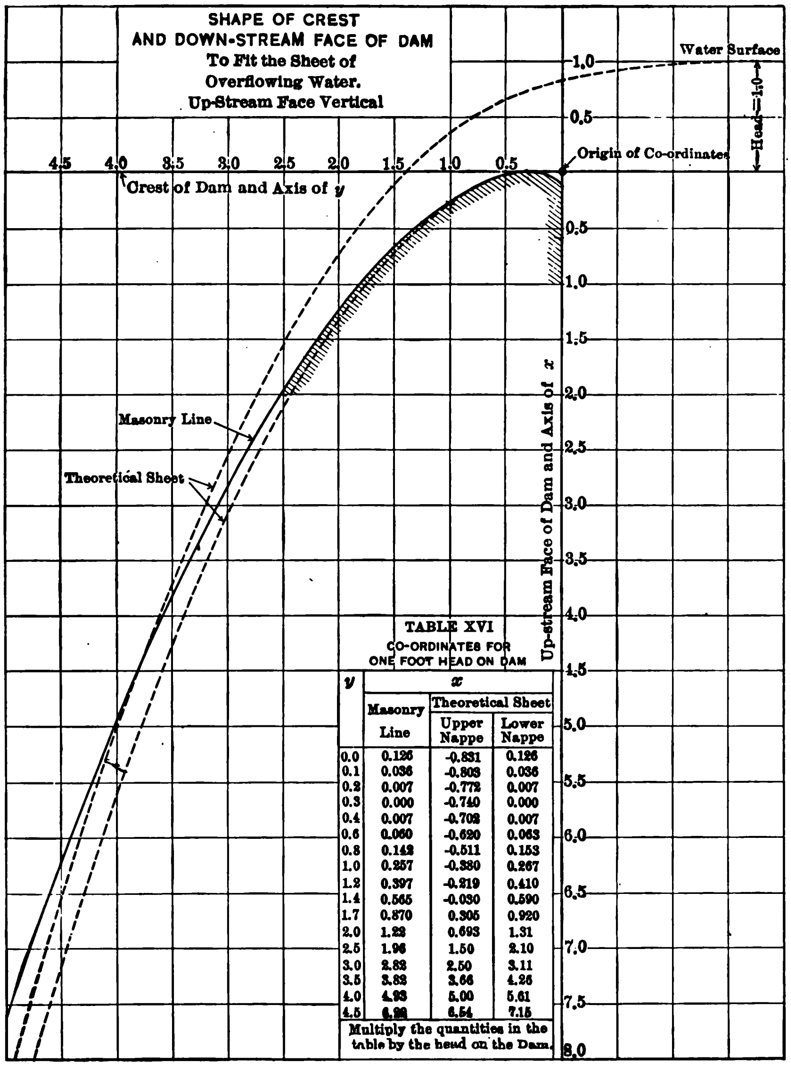


TABLE XVI

CO-ORDINATES FOR ONE FOOT HEAD ON DAM

y	x		
	Masonry Line	Theoretical Sheet	
		Upper Nappe	Lower Nappe
0.0	0.126	-0.831	0.126
0.1	0.036	-0.803	0.036
0.2	0.007	-0.772	0.007
0.3	0.000	-0.740	0.000
0.4	0.007	-0.702	0.007
0.6	0.060	-0.620	0.063
0.8	0.142	-0.511	0.153
1.0	0.257	-0.380	0.267
1.2	0.397	-0.219	0.410
1.4	0.565	-0.030	0.590
1.7	0.870	0.305	0.920
2.0	1.22	0.693	1.31
2.5	1.96	1.50	2.10
3.0	2.82	2.50	3.11
3.5	3.82	3.66	4.26
4.0	4.93	5.00	5.61
4.5	6.22	6.54	7.15

Multiply the quantities in the table by the head on the Dam.

FIG. 25.

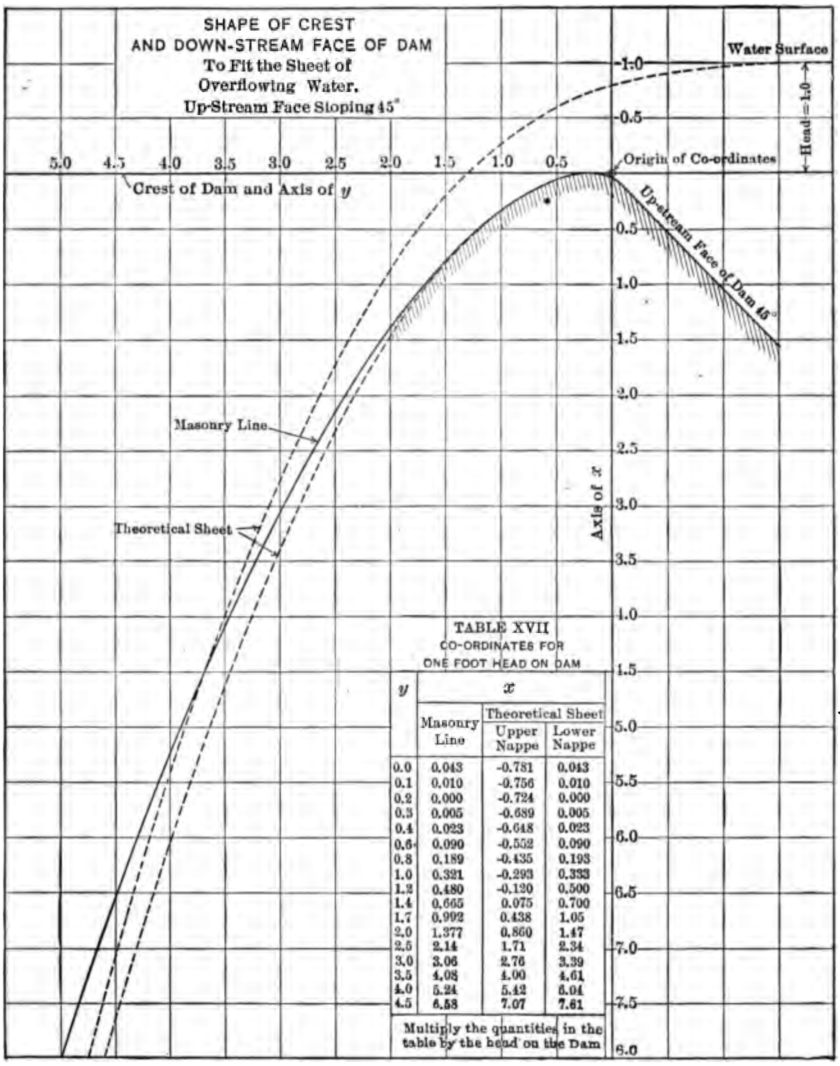


TABLE XVII
CO-ORDINATES FOR ONE FOOT HEAD ON DAM

y	x		
	Masonry Line	Theoretical Sheet	
		Upper Nappe	Lower Nappe
0.0	0.043	-0.781	0.043
0.1	0.010	-0.756	0.010
0.2	0.000	-0.724	0.000
0.3	0.005	-0.689	0.005
0.4	0.023	-0.648	0.023
0.6	0.090	-0.552	0.090
0.8	0.189	-0.435	0.193
1.0	0.321	-0.293	0.333
1.2	0.480	-0.120	0.500
1.4	0.665	0.075	0.700
1.7	0.992	0.438	1.05
2.0	1.377	0.860	1.47
2.5	2.14	1.71	2.34
3.0	3.06	2.76	3.39
3.5	4.08	4.00	4.61
4.0	5.24	5.42	6.04
4.5	6.58	7.07	7.61

Multiply the quantities in the table by the head on the Dam

FIG. 26,

where A is the thickness, 6–7. Values of q for the two types of crests, calculated in this manner, are given in Table XVIII. It is surprising to note that the calculated discharge agrees almost exactly with the discharge found by independent experiments for the same types of weirs.

The thickness of the sheet corresponding to any value of x' may now be derived from Eq. (30), the values of v_h and q being taken from Table XVIII. The area of the sheet lies one-third below and two-thirds above the curve.

In this manner the path of the falling sheet was determined and plotted in Figs. 25 and 26. The paths are only approximate, as they are extended from experimental points relatively close to the top of the dam, and, moreover, they may be somewhat affected by irregularities in the masonry crest. Consequently, it is advisable to provide a margin of safety by extending the masonry line well into the theoretical sheet as indicated. This line is usually formed by a series of arcs of circles, as indicated in Fig. 36.

The experiments were made with a negligible velocity of approach. The effect of velocity of approach on the shape of the sheet not being known, the best that can be done is to let h, Fig. 24, include the head corresponding to the velocity of approach and to consider that, when the velocity of approach is large, the results so obtained are correspondingly uncertain.

TABLE XVIII

FACTORS IN THE DETERMINATION OF THE SHAPE OF THE CREST. ($h = 1.0$ FT.)

Factor.	Water Face Vertical.	Water Face Inclined 45°.
a	+0.261	+0.249
b	−0.063	+0.007
v_h	6.63	6.52
y'^2	$2.732x'$	$2.640x'$
q	3.90	3.94

43. Discharge Capacity.* Francis has determined that the

* A very complete discussion of the theory and experiments relating to the discharge of water over dams may be found in U. S. Water-supply and Irrigation Paper No. 200, by R. E. Horton, and should be read by all those interested in the subject.

discharge of water over dams may be expressed by the following equation:

$$Q = ql_n = Cl_n\{(h_c+h_v)^{3/2} - h_v^{3/2}\}, \quad . \quad . \quad . \quad . \quad (31)$$

where, Q = the total discharge, in cubic feet per second;

q = the discharge per linear foot of effective crest;

l_n = the net or effective length of crest, i.e., the total length of crest corrected for end contractions due to piers and sharp-cornered abutments;

h_c = the actual or measured head on the crest, taken at a point sufficiently remote from the dam to avoid the surface curve;

h_v = the head corresponding to the velocity of approach; and

C = a coefficient which depends on the shape of the crest and the head on the crest.

In determining the head on the crest corresponding to the maximum flood to be expected, the following approximate equation may be used; provided the head on the crest is not greater than the depth of the channel of approach, the error for that head being much smaller than the error to be expected in determining the maximum flood:

$$Q = ql_n = Cl_n(h_c+h_v)^{3/2}. \quad . \quad . \quad . \quad . \quad . \quad (31a)$$

Francis' equation for the necessary correction due to complete end contractions is,

$$l_n = l_t - 0.1nh_c,$$

where, l_t = the total or gross length of crest between abutment and piers (Fig. 27), and

n = the number of complete end contractions.

If the crest is obstructed by piers having considerable widths and sharp corners, as indicated in Fig. 27, n represents the number of corners which serve to deflect the water, there being six complete contractions in this instance. Usually, however, the piers are relatively thin and are provided with sharp up-stream ends, as indicated in Fig. 28. In such cases the contractions for the piers are not complete, and Francis' equation would give values of l_n

too small. The number of experiments has not been sufficient to determine closely the effect of piers on the effective length of crest.

Unless the piers are unusually thin, relative to the head on the crest, or very considerably pointed up-stream, they may be con-

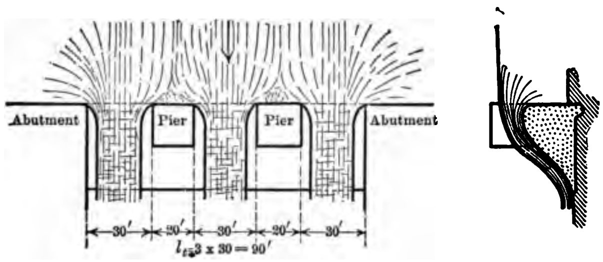


FIG. 27.—Complete Contractions Due to Abutments and Large Piers.

sidered to offer a partial contraction, probably amounting to not more than 0.04 h_c for each contraction, in case the piers are pointed as indicated in Fig. 29, and varying between this limit and 0.1 h_c for thick, blunt piers, depending on the degree of sharpness and the relative thickness.

FIG. 28.—Partial Contractions Due to Sharp Piers.

Francis' contraction equation may then be written,

$$l_n = l_t - h_c(C_a n_a + C_b n_b \ . \ . \ . \ C_n n_n), \quad . \ . \ . \ (32)$$

where C_a, C_b etc., represent the contraction coefficients applicable to the several different contractions which may be expected, and

n_a, n_b etc., the number of contractions having contraction coefficients, C_a, C_b, etc., respectively.

Thus, if the piers in Fig. 28 are shaped as indicated in Fig. 29, the effective length of crest would be,

$$l_n = l_t - h_c(0.1\times 2 + 0.04\times 4) = l_t - 0.36h_c.$$

Obviously, the effective length of crest between any two piers of whatever shape or, in the absence of piers, between the abutments, can never be less than 0.788 of the clear length of crest, as this reduction corresponds to the contraction of the width of a jet through an orifice in a thin plate.

The head corresponding to the mean velocity, v_1, in the channel of approach is $\frac{v_1^2}{2g}$.

The velocity in the channel of approach, however, is not uniform, the filaments above the elevation of the crest of the dam sometimes having a velocity considerably greater than the mean, depending on the depth and width of the channel, and its surface conditions. The energy of the filaments above the elevation of the crest have a proportionately greater effect in increasing the discharge. The true value, h_v, may be represented by

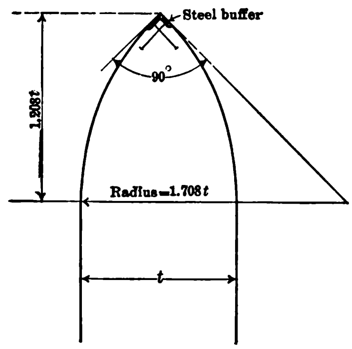


FIG. 29.—Typical Sharp Nose Pier.

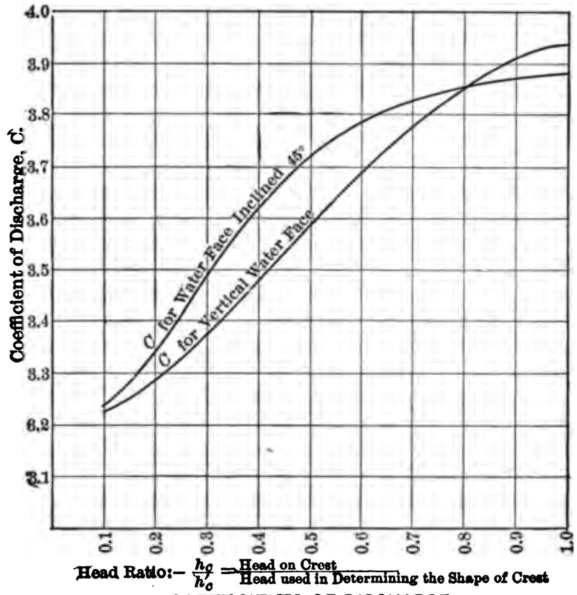
$$h_v = C_v\frac{v_1^2}{2g},$$

where C_v depends on the condition of the channel of approach described above. Values of C_v usually adopted in experiment work vary from 1.0 to 1.5.

However, in view of the fact that the velocity of approach above the elevation of the crest may be materially affected by wind, ice, and other conditions, it would seem advisable to use a value of $C_v = 1.0$, particularly as such assumption is on the safe side, when determining the capacity of the dam to pass the maximum flood.

Values of C, for Eqs. (31) and (31a), applicable to standard dam crests, may be taken from Fig. 30 which, in the absence of authentic experiments on this type of crest, was derived by the author from theoretical considerations and a comparison of a number of experiments on similar shapes of crests.

The values of the coefficients, for the head ratio of unity, are probably within 3 per cent of the truth; but, at other points on



COEFFICIENTS OF DISCHARGE
FOR STANDARD DAM CRESTS

FIG. 30.

the curves, the probable error is larger, increasing to perhaps 8 per cent for the smaller head ratios.

The head ratio of unity corresponds to the maximum flood to be expected, as the crest should be designed for that condition.

As h_v is a function of Q and h_c; and l_n a function of h_c, Eqs. (31) and (31a) cannot be solved directly, and successive substitutions of approximate values of h_v and l_n must be made. One or two trials will usually result in a solution sufficiently accurate for all practical purposes.

An example of the application of the foregoing equations is given in Example 5 of Art. 47.

If the crest of the dam is submerged, as in Fig. 31, the discharge coefficient, for use in Eq. (31) and (31a), should be modified according to the degree of submergence, as indicated in Table XIX.* In this table, C is the coefficient for *free* discharge over a similar crest under the same head, and C' is the modified coefficient due to the submergence. The heads are h_s and h_c, as in Fig. 31.

It will be noted that, for values of $\frac{h_s}{h_c}$ less than 0.30, the reduc- in discharge is less than 3 per cent.

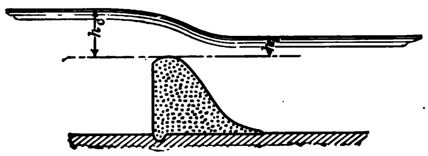


FIG. 31.—Submerged Crest.

TABLE XIX

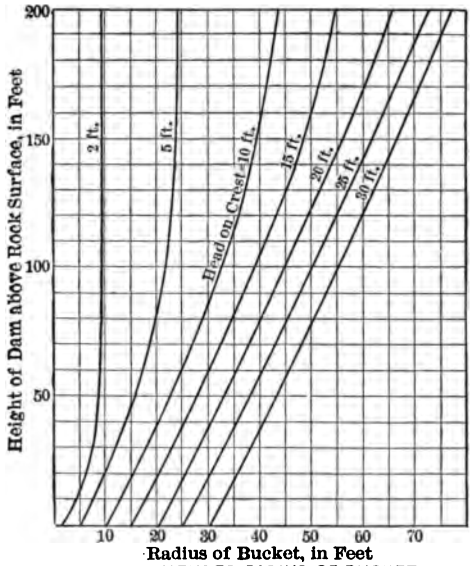
RELATIVE COEFFICIENTS, SUBMERGED CREST AND FREE CREST

$\frac{h_s}{h_c}$	$\frac{c'}{c}$	$\frac{h_s}{h_c}$	$\frac{c'}{c}$
0.0	1.000	0.5	0.937
0.1	0.991	0.6	0.907
0.2	0.983	0.7	0.856
0.3	0.972	0.8	0.778
0.4	0.956	0.9	0.621
		1.0	0.000

44. The Bucket. Except for low dams, small maximum discharges, and the best hard rock foundations, a fillet or " bucket " should be provided at the toe of the dam to deflect the sheet of water to a horizontal direction. The usual type of bucket is indicated in Fig. 4. Its use is obviously to prevent the impact of the falling water from scouring the foundation at the toe of the dam.

* From U. S. Deep Waterways experiments. See U. S. Water-supply and Irrigation Paper, No. 200, page 146. These experiments were made on a model having a rounded crest, approximating more closely than any of the others to the shape of a standard dam crest.

Although a parabola has been used in some instances for the curvature of the bucket, an arc of a circle has been adopted almost invariably, the former being an unnecessary refinement to establish a gradual transition. The proper radius of the bucket depends on the thickness of the sheet of falling water at the toe of the dam, the height of the dam, the character of the foundation, the quantity of débris and ice to be passed, the frequency of floods, and the depth of tail-water. For usual conditions, without tail-water, radii corresponding to those indicated in Fig. 32 are recommended.



RECOMMENDED RADIUS OF BUCKET
FOR SPILLWAY DAMS WITH NO TAIL-WATER

FIG. 32.

The diagram is purely empirical, and is based on the author's judgment and a comparison of a number of existing dams. The velocity of the water, as well as the thickness of the sheet, is affected materially by friction for dams having a large ratio of height to head on the crest. For instance, in the construction of the diagram, it was considered that the thickness and velocity of the sheet for a 2-ft. head on the crest would be practically constant for heights greater than 50 ft.

When there is some question as to the necessity of a bucket, its

construction may be deferred until such time as its need becomes apparent; providing that the toe of the dam is not submerged, rendering it accessible only by coffer-dams. For a submerged toe, the tendency is to neglect inspection, and the omission of a bucket, under such conditions, may affect the future safety of the structure.

The bucket, when used, is an addition to the predesigned section, and is not considered in the design.

45. Example No. 3. 91-ft. Solid Spillway Dam, Without Ice Pressure (Fig. 33). Assumptions:

H = maximum height of dam = 91 ft.;

c = the area of joints and base subjected to uplift = 50 per cent. The uplift is assumed to vary uniformly from head-water pressure at the heel to zero at the toe (there being for this example no tail-water); *

P_i = ice pressure = none;

h_c = maximum head on the crest = 10 ft.

Other assumptions as given for Example 1, Art. 38.

The first step in the design is to fix the shape of the crest and the upper part of the down-stream face, as described in Art. 42, to prevent the sheet of flowing water from leaving the face of the masonry. The origin of co-ordinates being taken on the prolongation of the up-stream face with Level 15. The co-ordinates of the masonry line and the theoretical sheet may be obtained by multiplying all quantities in Table XVI by 10, the maximum head on the crest, in feet.

As the considerations applicable to Zones I and I*a* have no influence on the shape of the section, they will be treated later.

The lower limit of Zone II, where the resultant, reservoir full, first intersects the down-stream extremity of the middle third can be found only by trial calculation, using Eq. (18), Art. 31.

In Zone III the slope of the down-stream face lies outside of the theoretical sheet of water in order to keep the resultant, reservoir full, within the middle third.

In Zone IV the resultants, reservoir full and empty, intersect the joints at the exact extremities of the middle third.

For the use of the general equations of design the reader is referred to Example 1, Art. 38. Results of calculations for Ex-

* See foot-note. p. 126.

ample 3 are given in Table XX. The maximum compressive stresses at the base are, for this example, well within allowed limits, and have no influence on the design.

Above the bottom of Zone I*a*, at the top of all spillway dams, the tangent of the angle of inclination of the resultant with the vertical is greater than the allowed value, being equal to the allowed value at the bottom of Zone I*a* and increasing to infinity at the crest of the dam, as indicated in Fig. 33. It is evidently impossible to provide sufficient weight of masonry to prevent sliding by friction alone and, above the bottom of Zone I*a*, monolithic concrete is generally used, although in the lower part of this zone, building joints inclined at an angle of $90° - \theta$ with the resultant would be acceptable. Key-ways, having a shearing value sufficient to resist the horizontal forces, may be used when it is desirable to have horizontal construction joints. It is permissible in many cases to increase considerably the allowed value of θ, within Zone I*a*, when a failure by sliding of the top of the dam would cause comparatively little damage, particularly if a considerable percentage of plums is used, which may be sometimes found to possess sufficient shearing and bearing value to resist the horizontal forces.

It will be noticed, in Fig. 33, that near the top of the dam (within Zone I), the resultant for full reservoir falls outside the middle third. This is also a condition common to all spillway dams, and can be met only by monolithic construction (if, as is usual, the tension is inconsiderable), or the use of steel reinforcement, as indicated in Art. 46.

46. Example No. 4. 87-ft. Solid Spillway Dam, with Ice Pressure (Fig. 34). This example is given in order to indicate the effect on the shape of the section, of the assumption of considerable ice pressure. For this example, ice pressure of 20,000 lb. per lin. ft. of dam is considered as acting at the elevation of the crest. Aside from ice pressure and tail-water, the assumptions for this example are the same as those for Example 3, Art. 45. It will be noted that there are two conditions of loading, as in Example 1, Art. 38.

First.—Low water, and ice pressure at the crest of the dam;
Second.—High water, and no ice pressure.

It is seen that, even with the greatly increased thickness which

TABLE XX

RESULTS OF CALCULATIONS FOR EXAMPLE NO. 3 OF ART. 45

Level.	Vertical Forces. Weight of Masonry $\Sigma(W)_E$.	Vertical Forces. Uplift Pressure.	Vertical Forces. Summation of Vertical Forces, Reservoir Full. $\Sigma(W)_F$.	Horizontal Forces. Head-water Pressure. $\Sigma(P)_F$.
55	140,800	28,200	112,600	75,080
65	198,200	40,100	158,100	109,400
76	273,000	56,400	216,600	154,400
90	386,800	81,200	305,600	222,400
106	542,700	115,500	427,200	315,900

Level.	Moments of Vertical Forces. Moment of Weight of Masonry $\Sigma(Wx)_E$.	Moments of Vertical Forces. Moment of Uplift Pressure.	Moment of Horizontal Forces. Moment of Head-water Pressure.	Total Moments. Sum of Moments of all Forces Reservor Full $\Sigma(Wx)_F+\Sigma(Px)_F$.
55	1,881,000	338,500	1,166,000	2,708,000
65	3,015,000	574,000	2,084,000	4,525,000
76	4,774,000	955,000	3,529,000	7,348,000
90	7,970,000	1,657,000	6,155,000	12,470,000
106	13,220,000	2,820,000	10,440,000	20,840,000

Level.	Location of Resultants. Distance of Resultant from Line of Reference. Empty Reservoir.	Location of Resultants. Distance of Resultant from Line of Reference. Full Reservoir.	Length of Base.	Distance of Heel from Line of Reference.	Tangent ϕ. Heel.	Tangent ϕ. Toe.
55	13.4	24.1	36.1	0	0	0.53
65	15.2	28.6	42.9	0	0	0.68
76	17.5	33.9	50.8	0	0	0.72
90	20.6	40.8	61.2	0	0	0.74
106	24.4	48.8	73.2	0	0	0.75

Level.	Tangent θ. Reservoir Full.	Maximum Pressures, in Thousands of Pounds per Sq. Foot. At Heel. Vertical.	At Heel. Inclined. Dam.	At Heel. Inclined. Foundation.	At Toe. Vertical.	At Toe. Inclined. Dam.	At Toe. Inclined. Foundation
55	0.667						
65	0.693						
76	0.713						
90	0.729						
106	0.739	14.8	14.8	14.8	11.7	18.3	18.1

has been adopted for the top of the dam, tension exists above Level 45.0 for the first condition of loading. The height of the portion of the dam in which tension exists may be reduced by increasing the thickness of the top of the dam. The proportions can be settled only by the judgment of the designer.

As it would be impracticable to provide monolithic concrete between the crest and Level 45, steel reinforcement must be used to resist the tension. The reinforcement should be computed in accordance with the theory of "flexure and direct stress,"* and a uniform uplift equal to head-water pressure should be assumed to act over the entire area up-stream from the neutral axis of any horizontal plane.

Fig. 35 indicates sections of the dam for water storage on the Upper St. Maurice River, Province of Quebec. The temperature of the site of the dam drops to -60 or $-70°$ F., and the range is about 160°. Ice pressure of 50,000 lb. per lin. ft. was assumed to act at crest level. The lines of resultants in the figure indicate clearly the need of reinforcement at the up-stream face.

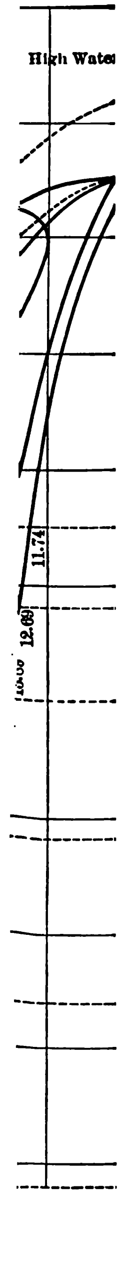
47. Example No. 5. 30-ft. Solid Spillway Dam (Fig. 36). Assumptions:

H = maximum height of dam = 30 ft. above good rock;
c = the area of joints and base subjected to uplift = 30 per cent. The uplift is assumed to vary uniformly from head-water pressure at the heel to tail-water pressure at the toe;†
w_1 = weight of masonry = 145 lb. per cu. ft.;
w_2 = weight of water = 62.5 lb. per cu. ft.;
w_3 = weight of silt in air = 125 lb. per cu. ft.;‡
α = angle of repose of silt in water = 0°;‡
K = voids in silt = 0 per cent;‡
f = safe value of the coefficient of friction of the joints and base = 0.75;
P_i = ice pressure = none;
Q_m = maximum flood to be expected = 32,400 cu. ft. per sec.;
l_t = total length of crest = 200 ft.;

* See "Principles of Reinforced Concrete Construction," by Turneaure and Maurer. 2d Edition. John Wiley & Sons. 1910.

† See foot-note, p. 126.

‡ Assumed as liquid mud. See Art. 16.



High Wate
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TABLE XXI

RESULTS OF CALCULATIONS FOR EXAMPLE NO. 4 OF ART. 46

Level.	Vertical Forces.			Horizontal Forces.			Loading.
	Weight of Masonry $\Sigma(W)_E$.	Uplift Pressure.	Summation of Vertical Forces Reservoir Full $\Sigma(W)_F$.	Ice Pressure.	Head-water Pressure.	Summation of Horizontal Forces Reservoir Full $\Sigma(P)_F$.	
45	108,700	16,540	92,160	20,000	28,120	48,120	1
52	145,900	22,050	123,900	20,000	42,760	62,760	1
60	192,200	29,330	162,900	20,000	63,250	83,250	1
72	269,700	49,700	220,000	....	137,100	137,100	2
86	375,600	71,900	303,700	....	201,900	201,900	2
102	520,300	103,200	417,100	....	290,900	290,900	2

Level.	Moments of Vertical Forces.		Moments of Horiziontal Forces.		Total Moments.	Loading.
	Moment of Weight of Masonry, $\Sigma(Wx)_E$.	Moment of Uplift Pressure.	Moment of Ice Pressure.	Moment of Head-water Pressure.	Sum of Moments of all Forces Reservoir Full, $\Sigma(Wx)_F + \Sigma(Px)_F$.	
45	1,476,000	193,500	600,000	281,200	2,164,000	1
52	2,160,000	280,700	740,000	527,000	3,146,000	1
60	3,083,000	407,700	900,000	949,000	4,524,000	1
72	4,803,000	786,000	....	2,944,000	6,961,000	2
86	7,569,000	1,360,000	....	5,305,000	11,514,000	2
102	12,110,000	2,345,000	....	9,208,000	18,970,000	2

Level.	Location of Resultants. Distance of Resultant from Line of Reference.		Length of Base	Loading.
	Empty Reservoir.	Full Reservoir.		
45	13.58	23.49	35.23	1
52	14.79	25.39	38.08	1
60	16.04	27.78	41.67	1
72	17.82	31.65	47.47	2
86	20.20	37.90	56.80	2
102	23.31	45.52	68.20	2

l_n = effective length of crest = l_t minus the effect of two complete end contractions;

l_c = width of the channel of approach = 210 ft.

MAXIMUM HEAD ON THE CREST

The velocity of approach, from Fig. 36, is

$$v_1=\frac{Q_m}{l_c(h_c+8)}=\frac{32{,}400}{210(h_c+8)}=\frac{154}{h_c+8}.$$

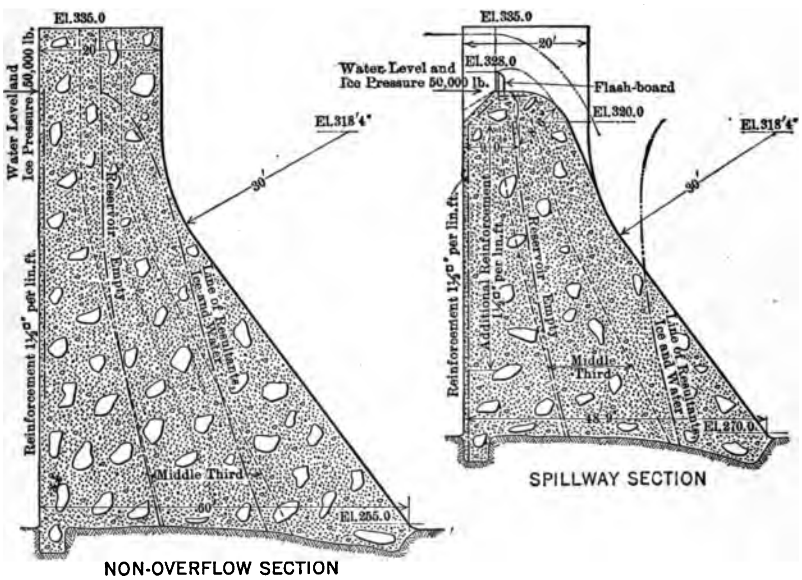


FIG. 35.—Upper St. Maurice River Dam, Showing Reinforcement to Resist Ice Pressure. (*Eng. Record*, Vol. LXX, p. 394.)

The head corresponding to the velocity of approach is

$$h_v=\frac{v_1^2}{2g}=\frac{1}{2\times 32.2}\left(\frac{154}{h_c+8}\right)^2=\frac{368}{(h_c+8)^2}.$$

As there are two complete end contractions, corresponding to two sharp-cornered abutments having crest coefficients of $C_a=0.1$, Eq. (32) may be written,*

$$l_n=l_t-h_c(0.1\times 2),$$

$$l_n=200-0.2h_c.$$

* The effect of end contractions, for this example, is negligible, but is included in order to indicate the application of Eq. (32).

The shape of the crest is to be proportioned to fit the sheet of water corresponding to the maximum flood to be expected; therefore the coefficient of discharge, from Fig. 30, will be $C=3.94$, corresponding to a head ratio of unity.

Substituting these values in Eq. (31a), there results,

$$32{,}400=3.94(200-0.2h_c)\left\{h_c+\frac{368}{(h_c+8)^2}\right\}^{3/2}.$$

This equation, being solved by successive trial substitutions, there results,

$$h_c=11.0 \text{ ft.},$$

whence

$$v_1=8.1 \text{ ft. per sec.},$$

$$h_v=1.0 \text{ ft.},$$

$$q=\frac{Q_m}{l_n}=164 \text{ cu. ft. per sec.}$$

These values are indicated in Fig. (36). The head, h_c, is the actual pressure head on the crest, and should fix the water surface to be used in determining the static head-water pressures on the dam. The head, h_c+h_v, is the total head on the crest as affecting the discharge, and should be used, as hereinafter described, in fixing the shape of the crest. The curve of the upper nappe may be drawn approximately, as indicated, from the plotted curve to the actual water surface.

Impact of the Approaching Water

From Art. 14 it is seen that the unit pressure from the impact of the approaching water corresponds to approximately twice velocity head, or

$$\text{Unit pressure}=2w_2h_v=2\times62.5\times1.0=125 \text{ lb. per sq. ft.}$$

The pressure is assumed to be distributed uniformly over the area above the silt.

Tail-water

The depth of tail-water at maximum flood is assumed to be 20 ft. From Art. 14, it is seen that the pressure of tail-water may or may not act upon the dam, depending on its depth relative to

the height of the dam, and the discharge. The determination of the depth, h_5, of tail-water which is apt to be swept away from the toe of the dam, as indicated in Fig. 4, is obtained from Eq. (5),

$$h_5 = \sqrt{\frac{q^2}{16.1h_3} + \frac{h_3^2}{4}} - \frac{h_3}{2},$$

where, q = the discharge per linear foot of crest, as previously determined;

and h_3 = the thickness of the sheet of water taken from Fig. 36, the shape of the sheet having been determined as hereinafter described. The thickness, h_3, should always be measured at the elevation of the toe of the bucket.

Making the proper substitutions, there results,

$$h_5 = \sqrt{\frac{164 \times 164}{16.1 \times 3.2} + \frac{3.2 \times 3.2}{4}} - \frac{3.2}{2} = 21.3 \text{ ft.}$$

This value, h_5, represents the minimum depth of tail-water which will resist the force of the falling water and remain at its full depth adjacent to the dam. Theoretically, a depth less than h_5 would be swept away, and that greater than h_5 would remain in contact with and press against the dam. It was explained in Art. 14 that, owing to the inexactness of Eq. (5), the dam should be tested for stability with and without tail-water, if, as in this example, the computed value, h_5, is within 20 per cent of the actual depth.

It will be seen that, in this example, the pressure of tail-water has a negligible effect on the location of the resultant. This condition, however, is not common to all cases.

SILT PRESSURE

From Eq. (8), Art. 16, the effective unit weight of submerged silt is

$$w_3 = w_3' - w_2(1-k),$$

where, w_3' = its unit weight in air;

w_2 = the unit weight of water; and

k = the percentage of voids.

Making the proper substitutions, there results,

$$w_3 = 125 - 62.5(1-0) = 62.5 \text{ lb. per cu. ft.}$$

The total pressure, P, above any horizontal joint may be found from Eq. (7),

$$P = \frac{w_3 h^2}{2}\left(\frac{1-\sin\alpha}{1+\sin\alpha}\right),$$

where h is the depth of silt above the joint and α its angle of repose. Therefore, since α is zero,

$$P = \frac{62.5h^2}{2} = 31.25h^2.$$

As explained in Art. 16, it is unusual to consider silt pressure of this kind and uplift to act at the same time. It is customary to test the dam for stability with uplift and before sedimentation, and also for no uplift and complete sedimentation. The two forces, however, are here assumed to act simultaneously in order to simplify the calculations.

Shape of the Sections

As previously indicated, the total head, $h_c + h_v = 12.0$ ft., should be used in fixing the shape of the crest and down-stream face. The proper shape of the crest and down-stream face to fit the sheet of spilling water may be obtained by multiplying all quantities in Table XVI by 12.0.

The masonry line having been laid out, the bucket may be added, as indicated in Fig. 36. The proper radius of the bucket for ordinary rock foundations may be taken from Fig. 32. A height of dam, above the bucket, of 28 ft. and a head on the crest of 12 ft. corresponds to an 18-ft. radius of bucket.

Test for Stability

The shape of the section having been adopted, it should be tested for stability, and altered if found necessary.

It is evident that the total length of bucket is probably insufficient to transmit the stresses to the foundations. The effective length of base may be determined by first calculating, from Eq. (28) of Art. 34, the maximum allowable inclination, ϕ, of the

down-stream face (Rule 4), it being usual to neglect that portion of the section lying outside of the line of maximum inclination, as indicated by the shaded area in Fig. 36:

$$\tan \phi = \frac{4}{3}f, \quad \text{or} \quad \sqrt{\frac{10}{H}}.$$

Making the proper substitutions, there results,

$$\tan \phi = \frac{4 \times 0.75}{3}. \quad \text{or} \quad \sqrt{\frac{10}{30}} = 1.0 \quad \text{or} \quad 0.577,$$

$$\tan \phi = 1.0, \text{ which is the greater of the two.}$$

Therefore $\phi = 45$ degrees.

The necessary calculations, including tail-water, for the determination of the stability of the dam above Level 42.0 will now be given. Calculations excluding tail-water may be made in the same manner.

The location of the resultant (Rule 1) may be determined from Eq. (18) of Art. 31. Using the subscripts, $_E$ and $_F$, to represent empty and full reservoir, respectively, referring to Fig. 13, and taking the point of reference on the up-stream face of the dam, we have,

$$z_E = \frac{\Sigma(Wx)_E + \Sigma(Px)_E}{\Sigma(W)_E},$$

$$z_F = \frac{\Sigma(Wx)_F + \Sigma(Px)_F}{\Sigma(W)_F}.$$

The necessary calculations are indicated in Table XXII.

Substituting in the foregoing equations,

$$z_E = \frac{1,359,000 + 0\ ^*}{102,600} = 13.24,$$

$$z_F = \frac{1,994,000}{93,190} = 21.4.$$

Plotting these values on Fig. 36, it is seen that the resultants lie well within the middle third.

* The pressure of silt is neglected for empty reservoir, representing the condition before the water first fills the reservoir.

TABLE XXII

TEST FOR STABILITY ABOVE THE BASE, FOR EXAMPLE NO. 5 OF ART. 47
(Tail-water included)

	Item	Factors.		Force.	Lever.		Moment.
Vertical Forces.	Concrete above level 42.0	Detailed calculations not given	$\Sigma(W)_E$	102,600	13.24	$\Sigma(Wx)_E$	1,359,000
	Vertical component of tail-water pressure	18.5 ×22 ×62.5 ×0.5		12,720	30.83		392,200
				115,320			1,751,200
	Uplift pressure from head-water	(41–22) ×62.5 ×37.5 ×0.5 ×0.3		–6,680	12.50		– 83,500
	Uplift pressure from tail-water	22 ×62.5 ×37.5 ×0.3		–15,450	18.75		–290,000
	Total uplift			–22,130			–373,500
			$\Sigma(W)_F$	93,190		$\Sigma(Wx)_F$	1,377,700
Horizontal Forces.	Silt pressure	31.25 ×22 ×22		15,120	7.33		110,700
	Head-water pressure	30 ×62.5 ×30 ×0.5		28,120	10.00		281,200
		11 ×62.5 ×30		20,620	15.00		309,300
	Impact pressure	8 ×125		1,000	26.00		26,000
				64,860			727,200
	Horizontal component of tail-water pressure	22 ×62.5 ×22 ×0.5		–15,130	7.33		–110,900
			$\Sigma(P)_F$	49,730		$\Sigma(Px)_F$	616,300
					$\Sigma(Wx)_F+\Sigma(Px)_F$		1,994,000

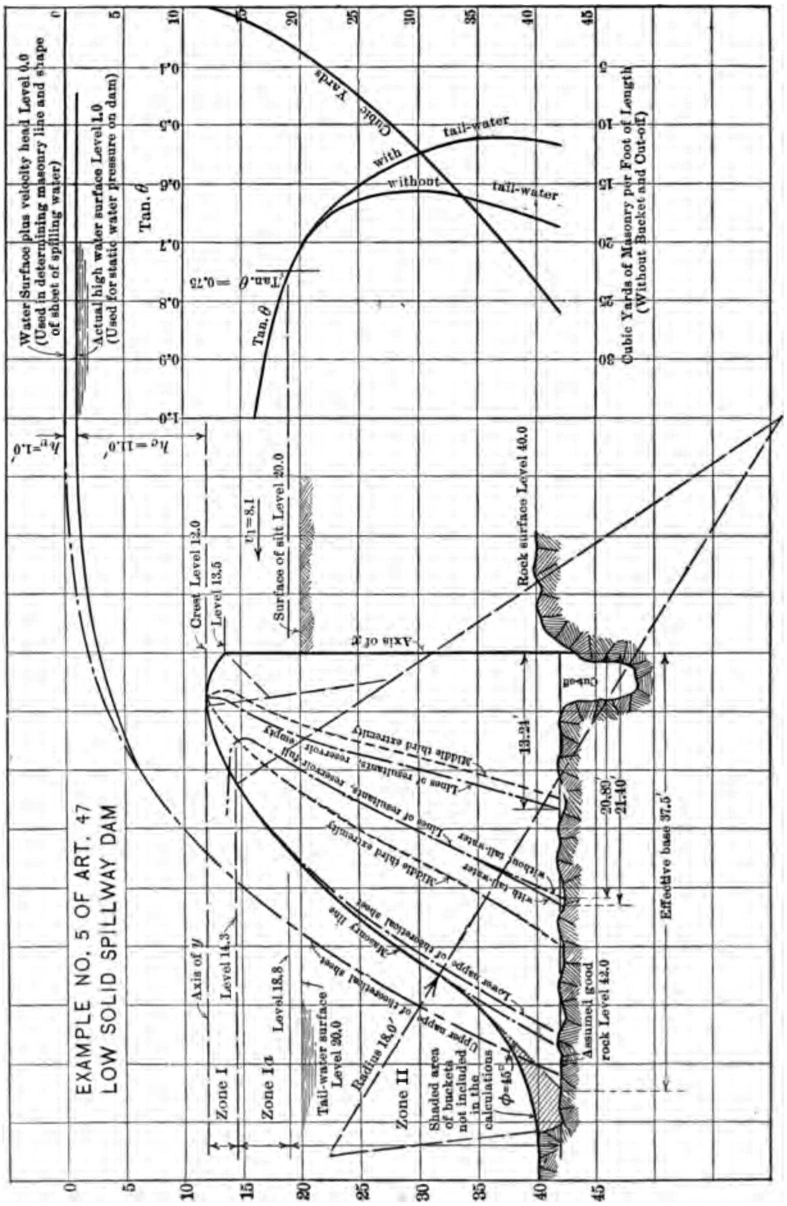

EXAMPLE NO. 5 OF ART. 47
LOW SOLID SPILLWAY DAM
Water Surface plus velocity head Level 0.0 (Used in determining masonry line and shape of sheet of spilling water)
Actual high water surface Level 1.0 (Used for static water pressure on dam)
Tan. θ
Cubic Yards
with tail-water
without tail-water
Tan. θ = 0.75
Cubic Yards of Masonry per Foot of Length (Without Bucket and Cut-off)
Crest Level 12.0
Level 13.5
Surface of silt Level 20.0
Rock surface Level 40.0
Axis of x
Axis of y
Level 14.3
Level 18.8
Tail-water surface Level 30.0
Zone I
Zone Ia
Zone II
Radius 18.0'
φ=45°
Shaded area of buckets not included in the calculations
Assumed good rock Level 42.0
Middle third extremity
Lines of resultants, reservoir full
Lines of resultants, reservoir empty
without tail-water
with tail-water
Masonry line
Lower nappe of theoretical sheet
Upper nappe of theoretical sheet
Cut-off
13.24'
20.80'
21.40'
Effective base 37.5'

FIG. 36.

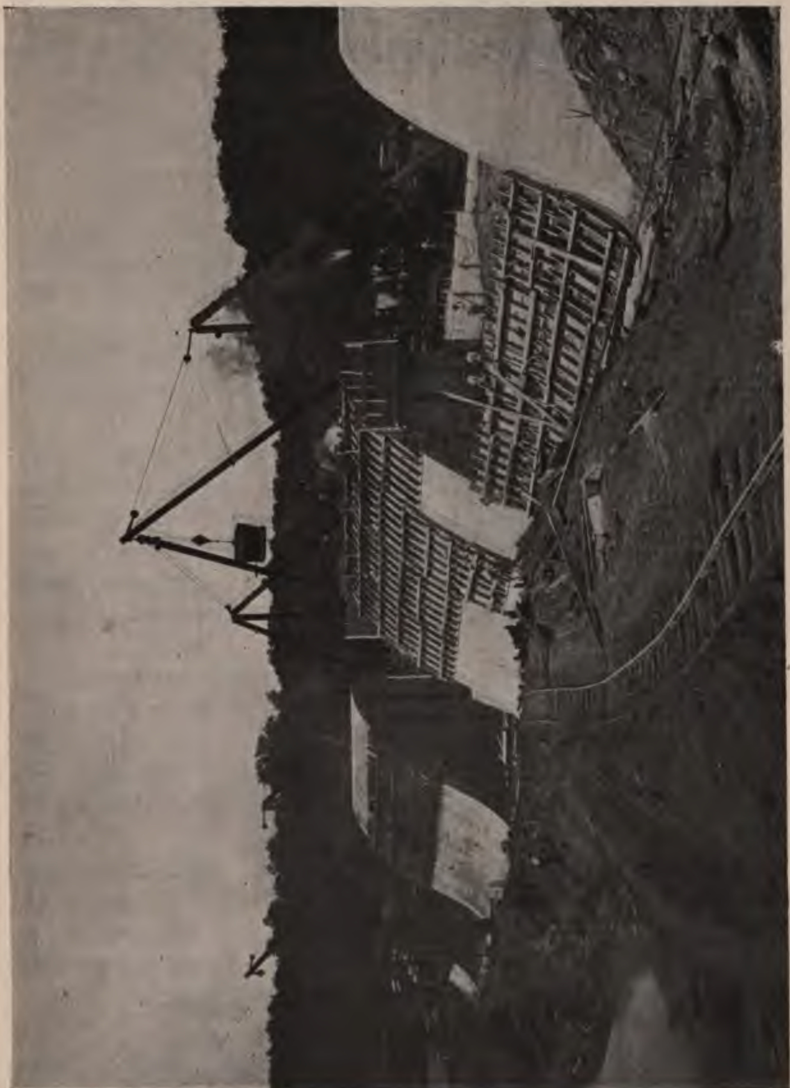


FIG. 37.—The Parr Shoals Dam on the Broad River, So. Carolina

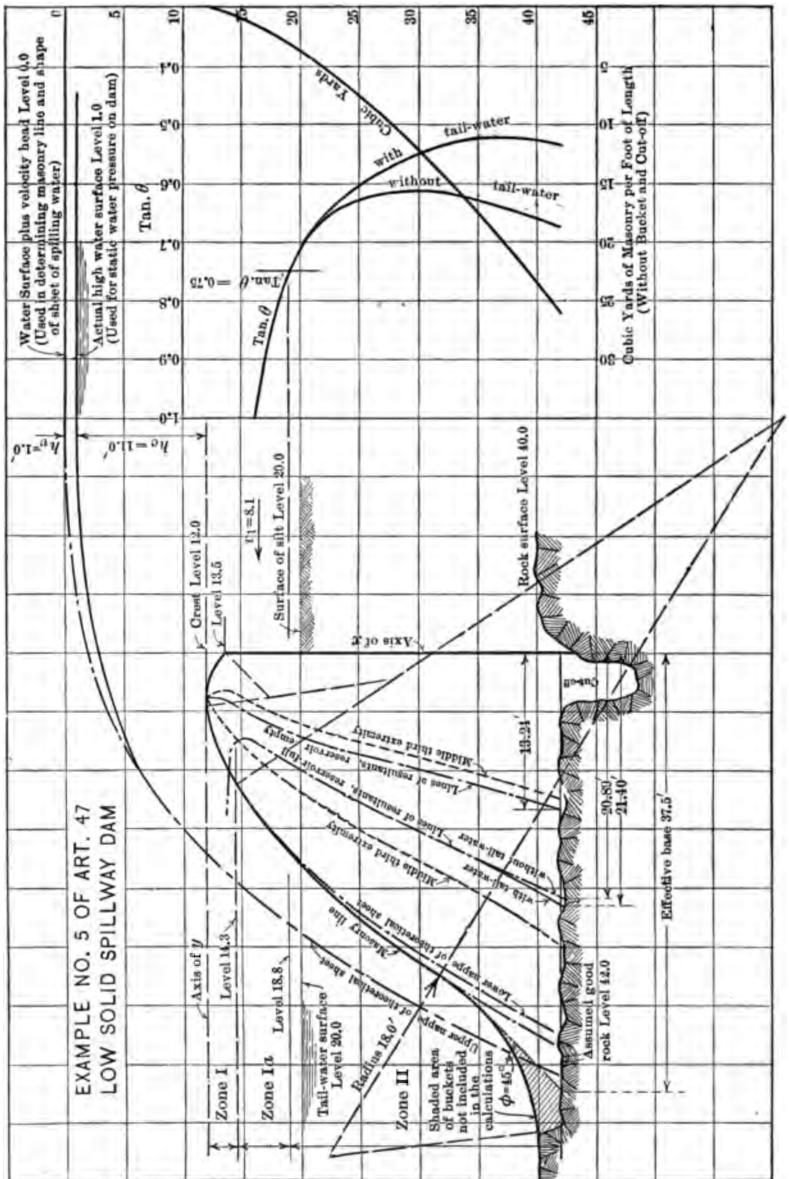

EXAMPLE NO. 5 OF ART. 47
LOW SOLID SPILLWAY DAM
Water Surface plus velocity head Level 0.0
(Used in determining masonry line and shape of sheet of spilling water)
Actual high water surface Level 1.0
(Used for static water pressure on dam)
Tan. θ
Tan. θ = 0.75
Cubic Yards
with tail-water
without tail-water
Cubic Yards of Masonry per Foot of Length
(Without Bucket and Cut-off)
h_a=1.0
h_c=11.0
Crest Level 12.0
Level 13.5
v_1=8.1
Surface of silt Level 20.0
Rock surface Level 40.0
Axis of x
Cut-off
Middle third extremity
Lines of resultants, reservoir full
Lines of resultants, reservoir empty
Middle third extremity
without tail-water
with tail-water
13.21'
20.80'
21.40'
Effective base 37.5'
Masonry line
Lower nappe of theoretical sheet
Upper nappe of theoretical sheet
Axis of y
Level 14.3
Level 18.8
Tail-water surface Level 20.0
Zone I
Zone Ia
Zone II
Radius 18.0'
Shaded area of buckets not included in the calculations
φ=45°
Assumed good rock Level 42.0

FIG. 36.

FIG. 37.—The Parr Shoals Dam on the Broad River, So. Carolina

To investigate for safety against sliding (Rule 2), we may use Eq. (22), of Art. 32. Evidently, Rule 2 is fulfilled for empty reservoir. For full reservoir, we have,

$$\tan \theta = \frac{\Sigma(P)_F}{\Sigma(W)_F} = \frac{49{,}730}{93{,}190} = 0.534,$$

which is well within the allowed value of 0.75.

Referring to Fig. 36, it is seen that the more severe condition, as affecting Rule 2, is without tail-water, although $\tan \theta$ is still well within the allowed limit.

Unit pressures in the masonry and the foundation (Rule 3), are obviously safe for this height of dam, but may be determined, if desired, as explained in Example 2 of Art. 39.

Results of calculations, for all elevations, and with and without tail-water, are indicated in Fig. 36. Above Level 18.8, (within Zone I*a*, Art. 30), $\tan \theta$ is greater than the allowed value; and above Level 14.3 (within Zone I), the resultant, reservoir full, lies outside the middle third. This condition necessitates careful treatment of the upper part of the dam, as described in Example 3 of Art. 45.

It will be noted that the shape of the section is governed solely by the shape of the sheet of falling water. In other words, the whole dam lies above the elevation of the bottom of Zone II. Should the calculations indicate a location of the resultant outside the middle third, the dam should be widened, as indicated in Example 3.

48. Comparison of Solid Spillway Dams. The fundamental theory of design, for solid spillway and non-overflow dams, differs only in the upper part, which, in the former, is proportioned to conform to the shape of the sheet of water spilling over the top. A comparison of this feature, therefore, would be of interest, and is given in Fig. 38.

In order to make a direct comparison with Example No. 5, the dimensions of each dam were multiplied by a constant, thus reducing each head on the crest to a value of 10 ft. This can properly be done, as it was shown in Art. 42 that the co-ordinates of the issuing sheet, and hence the curve of the top of the dam, are direct functions of the head on the crest. As a matter of fact, the

stability of any dam, subjected to head-water pressure only, will not be changed if all dimensions, including the head on the crest, are increased in the same proportion. This, of course, provided the compressive stresses remain within the allowed working values.

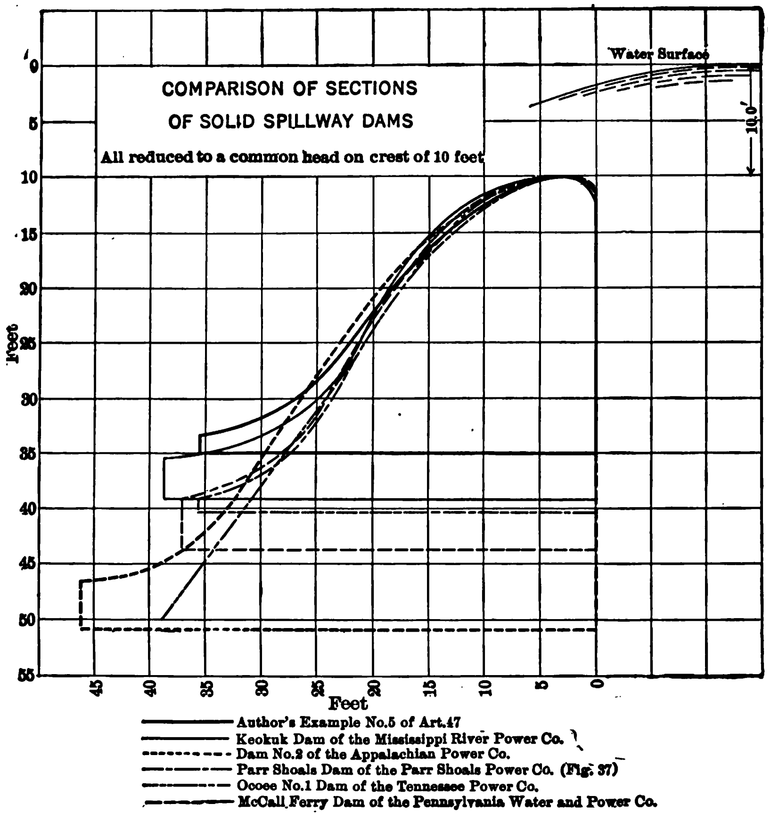


FIG. 38.

The differences in the tops of the dams indicated in Fig. 38 may be attributed chiefly to differences in the methods used in determining the shape of the sheet of spilling water and the distance the masonry line was extended into the sheet to provide a margin of safety.

CHAPTER VIII

THE DESIGN OF HOLLOW DAMS

49. General Considerations. Hollow dams may be made in an almost unlimited variety of forms; but, thus far, they have been largely constructed entirely of reinforced concrete, of the buttressed type, having substantially triangular buttresses set parallel to one another at fixed intervals across the bed of a stream.

Two types of hollow dams have become common. Fig. 47 indicates a typical " Ambursen " hollow dam. The distinguishing feature of this type is the flat reinforced concrete decks. Sometimes these decks are arched, as in Fig. 46. Such a small amount arching, however, will not be effective while the reinforcement is intact, as a shrinkage of the concrete due to setting, or a fall in temperature, will result in a slight opening of the joints and destroy all arch action until the reinforcement has failed. Therefore this feature should be considered as limiting complete failure, and not a partial failure, such as a cracking of the concrete and a rupture of the steel.

Fig. 43 indicates a typical " Multiple Arch " hollow dam in which the decks consist of series of arches spanning between the buttresses. The arches are often reinforced. For design of the arches see Art. 57.

The up-stream edges of the buttresses have a slope of any desired degree; but, in by far the larger number of cases, this slope is about 45°. The slope of the down-stream edges of the buttresses varies usually from zero to about 15° with the vertical for non-overflow dams. For spillway dams, the shape of the top and down-stream edge is usually fixed by the necessity of providing a crest and apron designed to fit the shape of the sheet of water spilling over the dam.*

The width and superelevation of the top is fixed by practical considerations, mentioned in Art. 37; but, in the case of hollow dams, a considerable superelevation or top width is not wholly or

* See Art. 42.

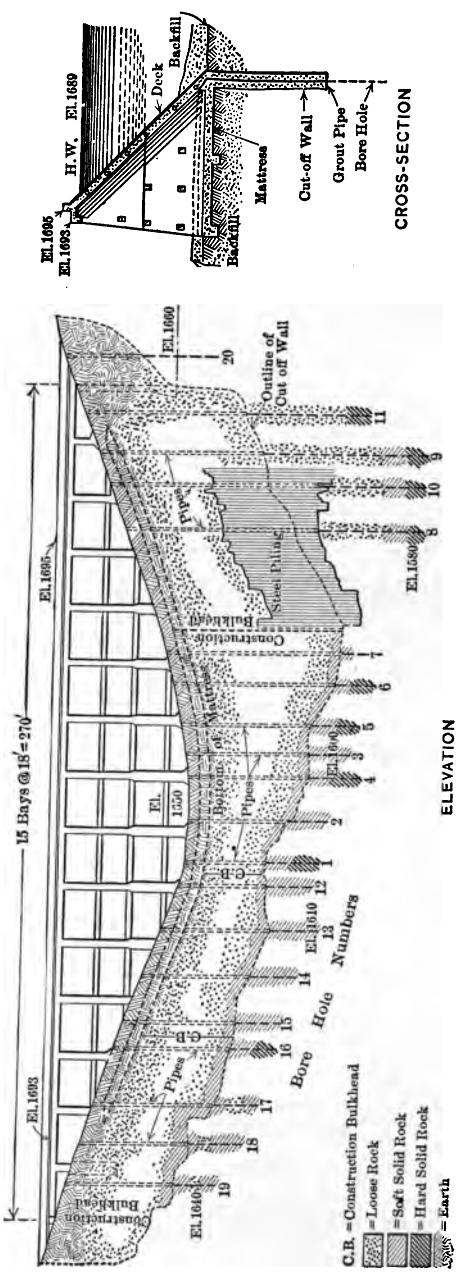


FIG. 39.—Elevation of Mathis Dike Dam and Foundations. (*Eng. News*, Vol. LXXIV, p. 591.)

partly compensated by a reduction in masonry in the lower parts, as in the case of solid non-overflow dams.

The apron of the spillway type is a reinforced concrete slab,

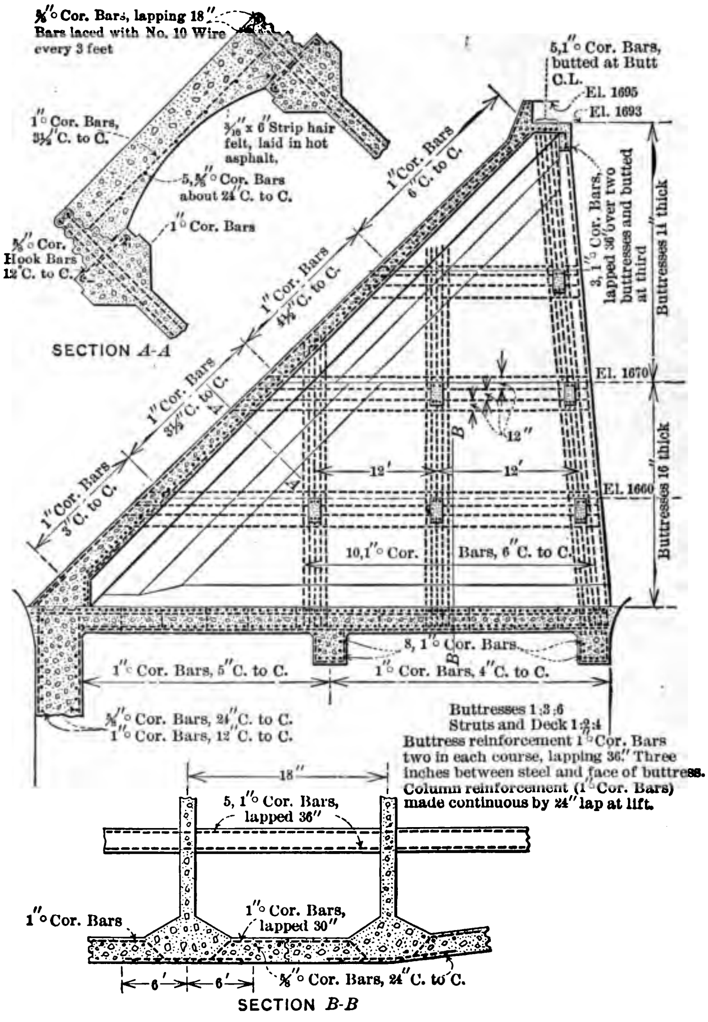


FIG. 40.—Details of Mathis Dike Dam. (*Eng. News*, Vol. LXXIV, p. 592.)

but, the loading being indeterminate, its thickness and reinforcing are matters of judgment. It is essential to provide ample concrete at the discharge lip, as indicated in Fig. 47, to withstand shocks

from floating ice and logs, and the pressure of the jet in changing its direction of flow.

On soft foundations the buttresses may be provided with plain or reinforced concrete spread footings, in order to keep the compressive stresses within reasonable amounts. In extreme cases, these footings may have a width equal to the spacing of the buttresses, so that the dam virtually rests on a concrete mattress, as indicated in Fig. 41. Such footings should be provided with large weep-holes at close intervals, in order to preclude the possibility of uplift from head-water.

FIG. 41.—Foundation Mattress, Mathis Dike Dam.

The buttresses should be braced with horizontal struts at intervals, as indicated in the illustrations. The spacing of the struts should not exceed twelve times the thickness of the buttresses when the latter are stressed in compression to their full working value. Additional stiffness is obtained by horizontal steel reinforcement between struts, as indicated.

The reinforcement in the struts is usually continuous through at least three bays, but in some cases it has been carried continuously throughout the structure, with no deleterious effects from contraction. The last method was used for the Mathis Dike Dam, Fig. 39, etc. The struts should abut solidly against the

buttresses. The horizontal building joints in the buttresses should be at the elevation of the center line of the struts, if possible.

The open holes through the buttresses, indicated in the illustrations, are found convenient for the passage of men and materials during construction. An inspection gallery is usually provided, unless, in low dams, access may be had from the down-stream side at ground level.

Provision for draining the interior of hollow spillway dams is usually made, as shown in Fig. 47, where, for this purpose, an opening of considerable size is indicated under the bucket. In

FIG. 42.—Down-stream View of Mathis Dike Dam.

such cases the high velocity of the water at the end of the bucket will entrain the air and cause a partial vacuum to form within the dam, unless sufficient air inlets are provided. If a partial vacuum is allowed to occur, the loads on the deck and apron will be materially increased, as explained in Art. 17. Large openings through the buttresses and an adequate open shaft at each end of the dam are usually provided. The mistake has sometimes been made of providing solid doors at the entrances to the interior of the dam.

In hollow dams of the usual type, the resultants for full or empty reservoir always intersect the joints relatively close to the

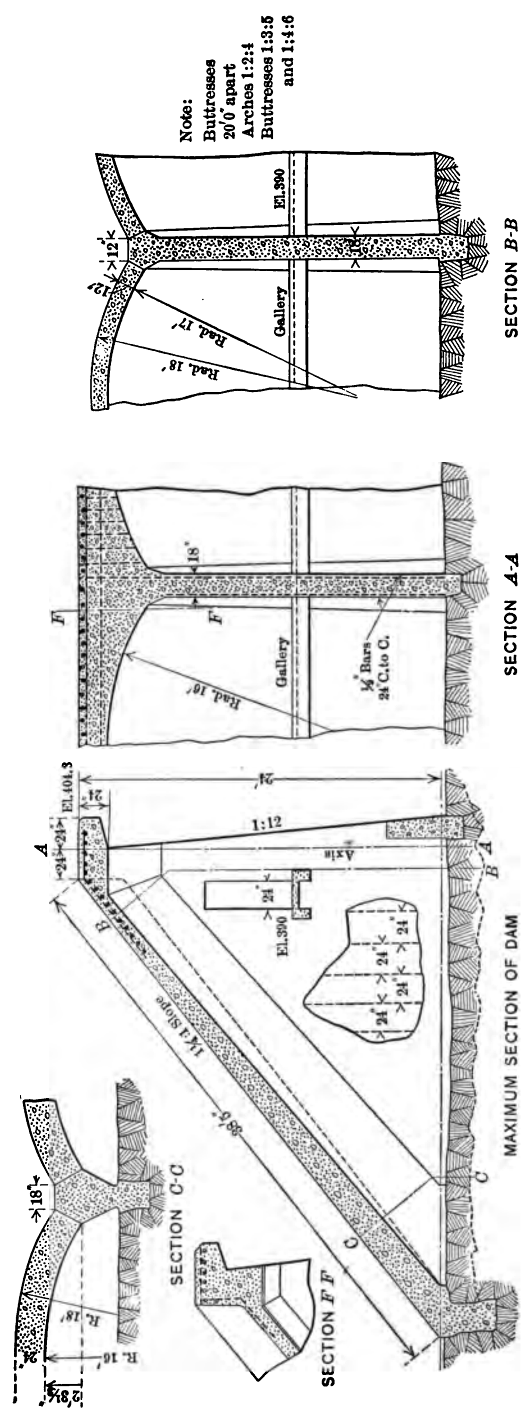


FIG. 43.—Multiple Arch Dam at Three Miles Falls, Umatilla Project, Oregon. (*Eng. News*, Vol. LXXIII, p. 1010.)

center of gravity. Rule 1, therefore, is seldom a governing consideration in determining the shape of the structure.

As the weight of masonry alone is never sufficient to prevent

FIG. 44.—Three Miles Falls Multiple Arch Dam.

sliding (Rule 2), it is necessary to batter the up-stream face to include the vertical component of considerable water pressure.

Limiting the compressive stresses to safe values (Rule 3) necessitates an adjustment of the length and thickness of the buttresses.

FIG. 45.—The Guyabal Hollow Dam, Porto Rico. (The Ambursen Company, New York.)

In hollow dams there is some question as to the effective area of base to distribute the pressures on the foundation. It is common practice to neglect the horizontal area of the deck in computing the compressive stresses. This procedure, however, is not always on the side of safety. The addition of the area of the deck to the area of the base will always reduce the direct stress (Fig. 12), but the eccentricity, e, may, in some cases, be increased a sufficient amount to increase the flexural stress to a greater extent, resulting in an increased total stress at the extremity of the base.

The complete design of hollow dams cannot be made, step by step, as in the case of solid dams. The general outlines of the dam are usually chosen tentatively, in accordance with the judgment of the designer, tested for conformity with the designing rules, and adjusted if found necessary. The spacing of buttresses is limited by the economical span of the deck. In general it will be found that a considerably greater spacing can economically be adopted for Multiple Arch Dams than for the Ambursen type. For large dams, a spacing of from 30 to 40 ft. for the former type and from 15 to 25 for the latter, are common.

In order to indicate more fully the several features entering into the design, reference is made to the examples which follow.

50. Example No. 6. Hollow Non-Overflow Dam (Fig. 46). Assumptions:

w_1 = weight of masonry = 150 lb. per cu. ft.;

w_2 = weight of water = 62.5 lb. per cu. ft.;

p_v' and p_v'' = maximum allowed vertical compressive stress = 31,000 lb. per sq. ft.;

p_i' and p_i'' = maximum allowed inclined compressive stress = 50,000 lb. per sq. ft.;

f = working value of the coefficient of friction of the joints and base = 0.60.

The section having been laid out, as indicated in Fig. 46, in accordance with the judgment of the designer, it remains to investigate it for conformity with the designing rules, and make the requisite alterations if found necessary. The methods will be described for the joint at Level 73.0 only. A length of dam of 18 ft. will be considered.

RULE 1. LOCATION OF THE RESULTANT

The location of the resultant may be found from Eq. (18) of Art. 31, moments being taken about the point of reference, A, at the up-stream extremity of the joint.

For empty reservoir,

$$z_E=\frac{\Sigma(Wx)_E+\Sigma(Px)_E}{\Sigma(W)_E}.$$

For full reservoir,

$$z_F=\frac{\Sigma(Wx)_F+\Sigma(Px)_F}{\Sigma(W)_F}.$$

The necessary calculations are indicated in Table XXIII.

Making the proper substitutions in the foregoing equations, there results:

For empty reservoir,

$$z_E=\frac{91{,}550{,}000}{1{,}943{,}000}=47.10.$$

For full reservoir,

$$z_F=\frac{235{,}450{,}000}{4{,}938{,}000}=47.71.$$

These values are indicated in Fig. 46.

RULE 2. THE INCLINATION OF THE RESULTANT

From Eq. (22) of Art. 32,

$$\tan\theta_F=\frac{\Sigma(P)_F}{\Sigma(W)_F}=\frac{2{,}916{,}000}{4{,}938{,}000}=0.591.$$

This value is close enough to the maximum allowed value of tan $\theta=0.60$, and no adjustment is necessary. The resultant, reservoir empty, is, of course, vertical.

A change in tan θ_F could best be obtained, if found necessary, by altering the inclination of the up-stream face of the dam, thereby changing most effectually the total force, $\Sigma(W)_F$, on account of the additional water pressure. It is essential, for economy, to have as steep an up-stream face as the allowed maximum value of tan θ_F will permit.

TABLE XXIII

CALCULATION FOR JOINT AT LEVEL 73.0. EXAMPLE NO. 6 OF ART. 50

Item.		Factors.		Force.	Lever.		Moment.
Vertical Forces.	Masonry above level 73.0	Details of calculations not given	$\Sigma(W)_E$	1,943,000	47.10	$\Sigma(Wx)_E$	91,550,000
	Vertical component of head-water pressure	72×74×62.5×0.5×18		2,995,000	24.67		73,900,000
			$\Sigma(W)_F$	4,938,000		$\Sigma(Wx)_F$	165,450,000
Horizontal Forces.	Horizontal component of head-water pressure	72×72×62.5×0.5×18	$\Sigma(P)_F$	2,916,000	24.00	$\Sigma(Px)_F$	70,000,000
					$\Sigma(Wx)_F+\Sigma(Px)_F$		235,450,000

In this connection, it should be noted that a much steeper upstream face is possible in the upper part of hollow dams; tan θ, for this example, being very small at the upper levels. A few recent hollow dams have been shaped in this manner.

RULE 3. COMPRESSIVE STRESSES

a. Reservoir Full. As the base of the dam is not rectangular, the vertical compressive stresses must be found by the method indicated in Art. 22 for irregular bases, and Eq. (14) and (15) will apply, p_u' and p_u'' being zero for hollow dams.

At the toe,

$$p_v' = \Sigma(W)_F \left\{ \frac{1}{A} + \frac{em'}{I} \right\},$$

At the heel,

$$p_v'' = \Sigma(W)_F \left\{ \frac{1}{A} - \frac{em''}{I} \right\}.$$

The following values for Level 73.0 are easily found:

$$A = 313.1;$$

$$I = 291{,}000;$$

$$e = 14.20;$$

$$m' = 62.71;$$

$$m'' = 33.29.$$

Using the value of $\Sigma(W)_F$ from Table XXIII, and making the proper substitutions in the foregoing equations, there results:

$$p_v' = 4{,}938{,}000\left(\frac{1}{313.1} + \frac{14.20 \times 62.71}{291{,}000}\right) = 30{,}870,$$

$$p_v'' = 4{,}938{,}000\left(\frac{1}{313.1} - \frac{14.20 \times 33.29}{291{,}000}\right) = 7{,}740.$$

The maximum vertical stress is seen to be at the toe of the dam, where it equals, approximately, the maximum allowed value adopted.

It will be found, upon investigation, that by properly reducing the thickness and increasing the length of the buttresses, and at

the same time slightly reducing the area of the joint, the vertical stress at the toe will remain the same and that at the heel will be increased. This adjustment is possible, owing to the resultant decrease in the eccentricity, e, of the loading, with a corresponding reduction in the flexural stress. On account of the decrease in the area of the joint without an increase in the maximum vertical stress, the resulting arrangement would be more economical. However, the thickness of the buttresses is limited by practical considerations, and, as it is considered that, in this example, they are as thin as it is advisable to have them, a more economical arrangement, in this respect is not possible.

For the maximum inclined compressive stresses in the dam, Eq. (23*a*), of Art. 33, applies. Eq. (24*a*) has no practical use unless the strength of the foundation is less than that of the masonry.

At the toe,

$$\tan \phi' = 0.333; \ \tan^2 \phi' = 0.111; \ \sec^2 \phi' = 1.153.$$

For full reservoir,

$$\tan \theta = 0.591; \ \sec^2 \theta = 1.352.$$

p_n is zero at the toe.

Using these values in Eq. (23*a*), the maximum inclined compressive stress for full reservoir is found to be:

At the toe

$$p_i' = (30{,}870 \times 1.153 - 0), \text{ or } 0, \text{ or } 30{,}870 \times 1.352,$$

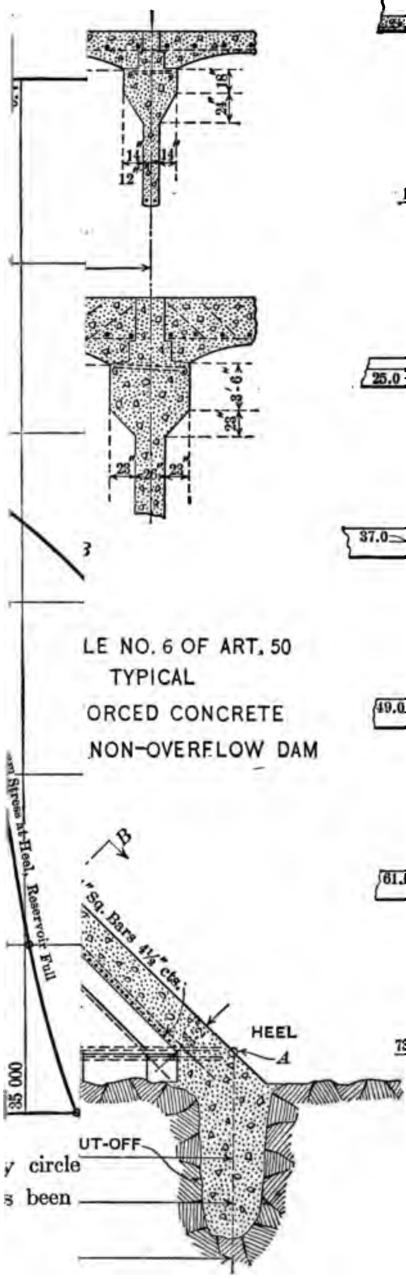
$$p_i' = 35{,}600 \text{ or } 41{,}750,$$

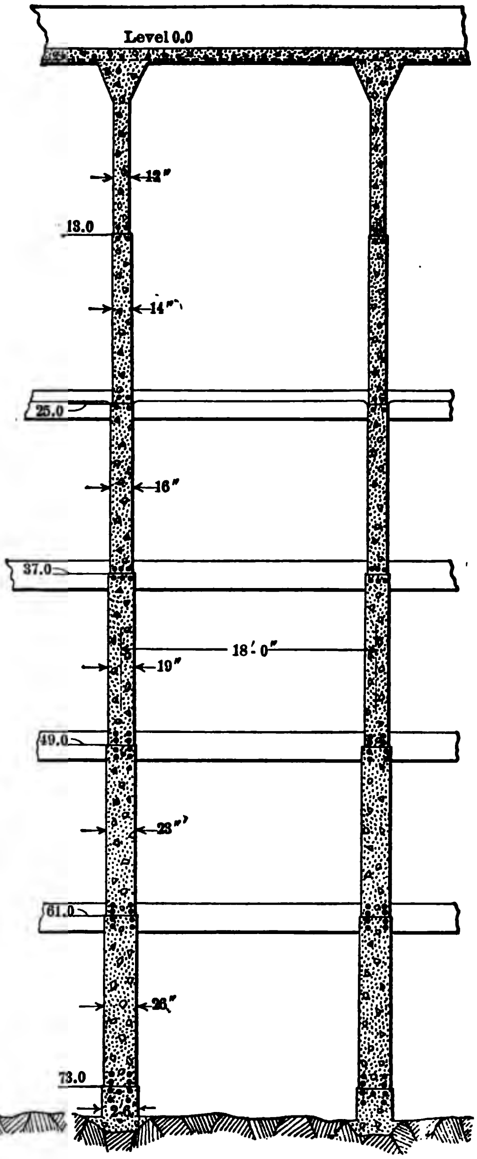
$$p_i' = 41{,}750, \text{ the greatest of these values.}$$

It is noted in Art. 33 that the greatest stress at the heel, for this type of dam, is the reaction of the deck on the buttresses. This will amount to,

$$p_i'' = \frac{62.5 \times 72 \times 18}{2.17} = 37{,}300.$$

The maximum inclined stress is seen to be within the adopted limiting value of 50,000 lb. per sq. ft.





SECTION *D-D*

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b. Reservoir Empty. As, for empty reservoir, both the eccentricity, e, and the weight, $\Sigma(W)_E$, is less than for full reservoir, it is evident that the stress, for this condition, will not govern.

The results of all these calculations are plotted in the diagram, Fig. 46, together with corresponding results for the upper joints. It will be noticed that the pressures grow rapidly less in the upper joints. To be consistent with theory, they should be constant at each joint. However, the thickness of the buttresses near the top of the dam is usually fixed by practical considerations. At the top of the dam a minimum thickness of 12 in. was considered advisable. At Level 49.0 a minimum thickness of 19 in. was required in order to keep within the allowed maximum vertical pressure of 31,000 lb. per sq. ft. Between these elevations the thickness of the buttresses was increased uniformly.

In order to indicate the difference in calculated stresses, with and without the horizontal area of the deck included in the area of the base, the vertical stresses, corresponding to the latter condition, have also been calculated and plotted in the diagram. It is seen that, at Level 73, to include the area of the deck, results in an increase in the maximum vertical stress from 23,600 to 30,870 lb. per sq. ft., and a corresponding increase in the maximum inclined stress. As mentioned in the last part of Art. 49, it is probable that the area of the deck is not wholly effective in distributing its portion of the loads, on account of not being directly bonded to the buttresses, and consequently the stresses may not be as high as calculated. It is certain, however, that they exceed those calculated without including the area of the deck, and it seems best to adopt the more conservative method.

RULE 4. INCLINATION OF DOWN-STREAM FACE

The maximum allowed inclination of the down-stream face is seldom a governing feature in the design of hollow dams. In the first place, it is never necessary to provide a very flat face, and in the second place, the usual horizontal reinforcement in the buttresses is in a position to resist possible tensile stresses in vertical planes.

51. Example 7. Hollow Spillway Dam. Fig. 47 represents a typical hollow spillway dam of the Ambursen type. In adopting

the general outlines, consideration should first be given to the shape of the crest and apron, which can be established at once, as described in Art. 42, Fig. 26 being used for the approximately 45° slope of deck. The radius of the bucket can be taken from Fig. 32.

The designing methods used in Example 5 apply directly to this case, and will not be repeated. The area of the horizontal joints should not include that of the apron. This is a necessary assumption; for, as indicated in Section *B–B* there is not a substantial bond between the apron and the buttresses. The assumption is on the side of safety.

Assumptions:

w_1 = weight of masonry = 150 lb. per cu. ft.;

w_2 = weight of water = 62.5 lb. per cu. ft.;

p_v' and p_v'' = maximum allowed vertical compressive stress = 29,000 lb. per cu. ft.;

p_i' and p_i'' = maximum allowed inclined compressive stress = 40,000 lb. per cu. ft.;

f = working value of the coefficient of friction = 0.60.

The necessary calculations for the determination of the location of the resultants, at Level 64.0, are indicated in Table XXIII.

The results of all calculations are plotted in the diagram of Fig. 47. It will be noted that the stresses vary considerably at the different levels, and approach the maximum allowed values only at Level 64.0. This could be remedied only by decreasing the thickness of the buttresses above Level 64.0. However, they are as thin as it is desirable to have them, so that no adjustment, in this respect, is possible.

Some economy would result if the slope of the up-stream face were steepened slightly, as it is seen that the maximum value of tan θ is less than the allowed value of 0.60.

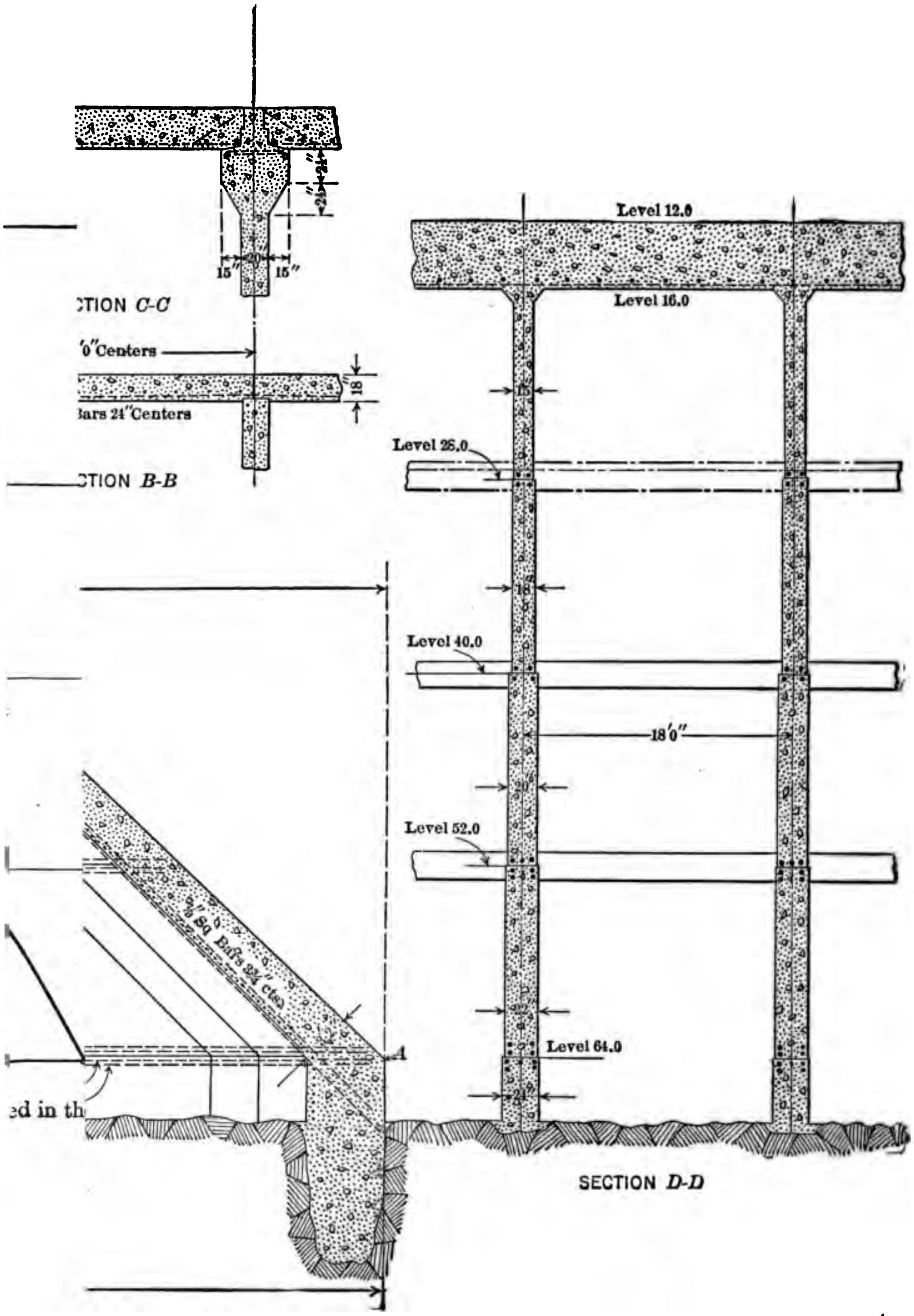

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TABLE XXIIIA

CALCULATIONS FOR JOINT AT LEVEL 64.0. EXAMPLE NO. 7 OF ART. 51

Moments about point "A," Fig. 47

Item.		Factors.		Force.	Lever.		Moment.
Vertical Forces.	Masonry above level 64.0	Details of calculations not given	$\Sigma(W)_E$	1,565,000	48.50	$\Sigma(Wx)_E$	75,930,000
	Vertical component of head-water pressure	52×55×0.5×62.5×18		1,609,000	18.33		29,490,000
		12×55×62.5×18		742,000	27.50		20,410,000
			$\Sigma(W)_F$	3,916,000		$\Sigma(Wx)_F$	125,830,000
Horizontal Forces.	Horizontal component of head-water pressure	52×52×0.5×62.5×18		1,521,000	17.33		26,380,000
		12×52×62.5×18		702,000	26.00		18,250,000
			$\Sigma(P)_F$	2,223,000		$\Sigma(Px)_F$	44,630,000
					$\Sigma(Wx)_F+\Sigma(Px)_F$		170,460,000

CHAPTER IX

ARCH DAMS

52. General Considerations. Arched dams may be divided into two general classes:

a. *Curved gravity dams*, or those designed as gravity dams, but curved in plan so as to make use of arch action as an additional margin of safety. Such dams may be designed in accordance with the theory hereinbefore given, and need no further discussion, as the arch action makes for a reduction in all stresses, and provides a more favorable location of the resultant, reservoir full.

b. *Arch dams*, or those for which the arch action provides most or practically all of the resistance to the forces tending to move the structure. This chapter will be limited to a discussion of this type.

In the discussion of arch stresses, the assumption will first be made that all the load is taken by arch action, the deflection of the dam being unrestrained by the shearing and frictional resistance at the foundation. The modification of the arch stresses due to restraint at the base will then be discussed.

The following nomenclature will be used for arch dams, all distance being in feet and unit pressures in pounds per square foot:

m = the deflection of the unrestrained arch at any elevation;
m_1 = the deflection of the restrained arch at any elevation;
M_1 = the deflection at the top of the restrained arch;
t = the thickness of the arch at any elevation;
h = the height of the dam above any elevation;
H = the total height of the dam;
r_u = the radius of the arch at any elevation measured to the upstream face;
r_m = the mean radius of the arch at any elevation;

q = unit load taken by the unrestrained arch at any elevation;
q_1 = the unit load taken by the restrained arch at any elevation;
p = the stress in the unrestrained arch at any elevation;
p_1 = the stress in the restrained arch at any elevation;
w_2 = the unit weight of water;
$C = \frac{C_1}{C_2}$ = all constants;
k, k', k'' = constants;
A = horizontal area of an arch dam, between abutments, at any elevation;
δ = central angle between abutments (Fig. 52);
s = span of arch at any elevation;
π = 3.1416.

53. Arch Stresses. In order to indicate the approximate distribution of stresses in an arch dam, we will first consider the simple case of a dam of constant radius, confined between vertical abutments, and having a horizontal base. It will be assumed that:

a. The arch radius is very long, compared with the thickness of the dam;
b. The dam is homogeneous throughout;
c. The thickness at any elevation is proportional to the depth of water, resulting in a triangular section with its crest at water surface;
d. The arch stresses are not influenced by temperature changes and other indeterminate considerations to be discussed later.

The shaded area, 1–2–3, in Fig. 49, represents a section of the unloaded dam at the crown of the arch. The unit water loads are represented by abscissæ to the line, 1–7.

Considering the deflection of the loaded dam to be unrestrained at the base, the ordinary equation for the arch stress at any elevation is

$$p = \frac{qr_u}{t}. \quad \ldots \ldots \ldots \quad (33)$$

This equation is not correct when the arch radius is small,

compared with the arch thickness; but it is close enough for all practical purposes.

FIG. 48.—Arch Dam of the Shoshone Project, Wyoming.

As $\frac{q}{t}$ and r_u are assumed to be constant for this case, p will also be constant, and will be represented by abscissæ to the line, 8–9.

The late R. Shirreffs has shown * that, for constant radius and span, the crown deflection of an unrestrained arch may be represented by,

$$m=\frac{C_1q}{t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

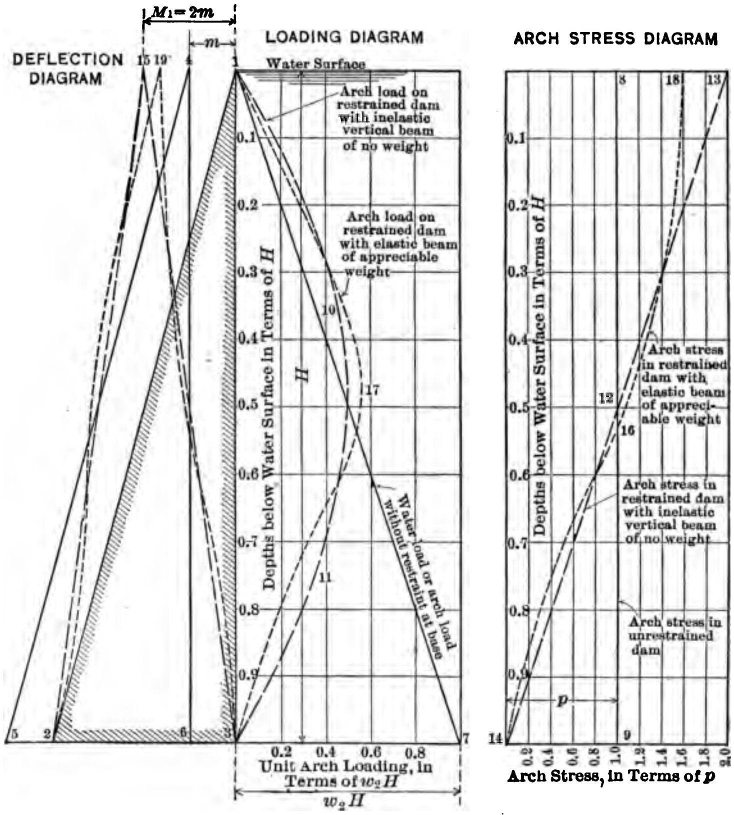


FIG. 49.

where C_1 is a constant depending on the radius, span, and other characteristics of the arch.

By assumption,

$$t=C_2h, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and,

$$q=w_2h,$$

* In his discussion of a paper on Lake Cheesman Dam by Harrison and Woodard, *Transactions*, Am. Soc. C. E., Vol. LIII, p. 155.

therefore,

$$m=\frac{C_1w_2h}{C_2h}=Cw_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

Therefore m at any elevation is constant, and the dam under load will assume the position, 4–5–6.

A restraint at the base, however, on account of vertical beam action, will result in a redistribution of the loading taken by the arch. If we assume the vertical beam to be incapable of deformation, of no weight, and free to rotate about the base, the load carried by the arch may be represented (as proved later), by the curve, 1–10–11–3, and the arch stress by the line, 13–14. The deflection of the dam will be as indicated by the lines, 15–2–3, the section still being a true triangle on account of the inelasticity of the vertical beam. The area between the vertical, 1–3, and the curve, 1–10–11–3, represents the total load now being carried by arch action, and is less than the total water load, 1–7–3–1, the remainder being taken directly to the foundation by shearing and frictional resistance at the base. All of this can be proved, as follows:

Shirreffs' equation applied to the restrained arch is,

$$m_1=\frac{C_1q_1}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34a)$$

Substituting the value of t from Eq. (35), there results,

$$m_1=\frac{C_1q_1}{C_2h}=\frac{Cq_1}{h}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

For static equilibrium, the moment, about the base, of the loading carried by restrained arch action must be equal to the corresponding moment of the total water load.

Therefore,

$$\int_0^H q_1(H-h)dh=\frac{w_2H^3}{6}. \quad . \quad . \quad . \quad . \quad . \quad (38)$$

From Eq. (37),

$$q_1=\frac{m_1h}{C}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

And since, by assumption, the up-stream face of the dam is still straight,

$$m_1=\frac{M_1(H-h)}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

Substituting these values of q_1 and m_1 in Eq. (38), there results,

$$\int_0^H \frac{M_1(H-h)^2h}{HC}dh=\frac{w_2H^3}{6}.$$

Integrating between the limits 0 and H, and solving for M_1 we have, as the top deflection of the restrained dam,

$$M_1=2Cw_2. \quad \ldots \ldots \quad (41)$$

Comparing the top deflection of the restrained arch, as given by Eq. (41), with that of the unrestrained arch, from Eq. (36), it is seen that the former is just twice the latter under the assumption of no deformation of the vertical beam.

To derive the equation of the loading curve, 1–10–11–3, we have, from Eq. (39), (40) and (41).

$$q_1=\frac{m_1h}{C},$$

$$m_1=\frac{M_1(H-h)}{H},$$

$$M_1=2Cw_2,$$

Therefore,

$$q_1=\frac{2w_2(H-h)h}{H}, \quad \ldots \ldots \quad (42)$$

which is the equation of the curve, 1–10–11–3.

The equation of the stress line, 13–14, may be derived as follows: At the base of the dam we have, from Eqs. (33), (35) and (42), for the restrained arch,

$$p_1=\frac{q_1r_u}{t}=\frac{q_1r_u}{C_2h}=\frac{2(H-h)w_2r_u}{HC_2}. \quad \ldots \quad (43)$$

Also, at the base of the unrestrained dam, Eq. (33) gives,

$$p=\frac{w_2Hr_u}{t}=\frac{w_2Hr_u}{C_2H},$$

or,

$$C_2=\frac{w_2r_u}{p}.$$

Substituting this value of C_2 in Eq. (43), there results,

$$p_1=\frac{2p(H-h)}{H}, \qquad (43a)$$

which is the equation of the stress line, 13–14.

It will be noted that, for the restrained dam, the arch stress, p_1, is equal to the constant unrestrained arch stress, p, when h is one-half of H. In the upper half of the dam the arch stress for the restrained dam is greater than that for the unrestrained dam, reaching a maximum of *twice* the latter value at the extreme top. In the lower half, the restrained arch stresses are less, reaching a minimum value at the base, where the arch deflection is zero.

Thus far we have been limited by assumptions, some of which are impossible and others of which are unusual. In order to arrive at a closer indication of the arch stresses, we must investigate the effect of:

a. Elasticity of the vertical beam;
b. The weight of the dam;
c. Uplift water pressure;
d. Varying span;
e. Varying radius;
f. Possible variation of the modulus of elasticity at different elevations;
g. Expansion and contraction due to temperature changes;
h. Expansion and contraction due to changes in moisture content;
i. Contraction of the concrete due to setting of the cement.

On account of the indeterminate effect of most of these items, it is the opinion of the author that it is impossible to write exact equations indicating the arch and vertical beam stresses in arch dams, although many attempts have been made.*

Fortunately, a number of dams have been constructed which, being virtually experiments on a large scale, serve to indicate

* *Transactions*, Am. Soc. C. E., Vol. LIII, p. 155, paper by Harrison and Woodard, and discussion; *Transactions*, Tech. Soc. Pacific Coast, Vol. VI, p. 75, paper by Vischer and Wagoner; *Transactions*, Am. Soc. C. E., Vol. LXXVIII, p. 685, paper by L. R. Jorgensen; *Engineering News*, June 9, 1910, article by J. S. Eastwood; "Masonry Dam Design," by Morrison and Brodie. 2d edition. John Wiley & Sons. 1916.

h_a	Material of the Dam.	Reinforcement.
(11)	(40)	(41)
	oncrete	None
0	oncrete	None
0	ubble masonry	None
0	oncrete	None
0	ubble masonry	None
0		
	oncrete	18 horizontal rows of 40-lb. rails in top 20 ft.
0	oncrete	None
0	oncrete	None
0	oncrete	$\frac{3}{4}''$ ϕ hor. rods 15″–18″ cts.; 12-lb. vert. rails 4′ 6″ cts. in each face
0	oncrete	$1\frac{1}{4}''$ hor. steel cables 2′ cts. on center line of dam.
0		
	oncrete	See reference.
0	oncrete	None
0	ubble masonry	None
0	oncrete	None
0	oncrete	None
0		
	oncrete	
0	oncrete	
0	oncrete	$\frac{3}{4}''$ hor. rods 2′ cts. in both faces, in top 40′ of dam
0	oncrete	None
0	oncrete	None
0		
	oncrete	None
0	oncrete	
0	oncrete	None
0	oncrete	None
0	oncrete	None
0		
	oncrete	20-lb. rails hor. and vert.—See text
0	oncrete	None
0	oncrete	None
0	oncrete	None
..0	oncrete	See reference
0		
	oncrete	
0	oncrete	None (Dam to be raised 14′)
0	oncrete	$\frac{3}{4}''$ ϕ hor. rods 12″ cts. in each face
0	oncrete	None (Ultimate height 50′)
0		

top of the dam. h_e is the total height of dam.
e to up-stream face of dam.
e to down-stream face of dam.
eam face of dam.
in a straight line between haunches of arch.
iformly between points indicated.
ction, temperature changes, and other indeter-

50, common to some existing dams, has been
red at the elevation of the top of the pedestal.
inished as an arch; where an enlarged base has
stal
h da to keep sheet of spillway water from leaving

To face page 155

g N

what may be expected from structures of this type. As far as the author is aware, there are no records of the failure of an arch dam. This, though fortunate from an economic point of view, is lamentable, as there has been furnished no direct indication of the limitations of such structures. Of the existing dams, a few have been designed in accordance with intricate although—in the author's opinion—questionable theory; but by far the greater number have been treated simply as sections of rigid cylinders subjected to external water pressure, *i.e.*, thickness at any elevation determined from Eq. (33), without consideration being given to possible restraint at the base.

Table XXIV gives the general characteristics of a number of representative arch dams. The arch stresses indicated in Column 9 were calculated from Eq. (33). In order to indicate the relative stability of future designs, in comparison with existing structures, there is given the following brief discussion of the general effect, on stability, of the items listed above.

Under the original assumptions, we have seen that, for an unrestrained arch dam, the arch stress may be represented by the line 8–9 (Fig. 49), and that, when the dam is restrained and the vertical beam is inelastic and weightless, the arch stress changes to 13–12–14.

The effect of the elasticity of the vertical beam is to decrease the deflection at the top of the dam and increase the deflection at lower elevations. The effect of the weight of the structure and a resistance to rotation at the base is to increase the proportion of the load carried directly to the foundation and to reduce the arch stresses at all elevations. The arch stress curve under these combined conditions will have the general shape indicated by the line, 18–16–14, the exact amount of stress at the different elevations being dependent on the relative influence of the imposed conditions.

The approximate loading diagram and the deflection of the dam corresponding to the arch stress curve, 18–16–14, are indicated by the lines, 1–17–3 and 19–2–3, respectively.

The effect of the uplift pressure due to head-water on horizontal joints tends only to lessen the effective weight of the dam and consequently the influence of the weight on the distribution of arch stresses. Uplift may be neglected in structures of this type, as it has practically no influence on stability.

Other things being equal, the smaller the span or the radius, the greater will be the load taken by an arch for a given deflection. Most sites are V-shaped, resulting in relatively shorter spans at the lower elevations; moreover, as will be shown later, the greatest economy of material results for V-shaped sites when the arch radius gradually decreases toward the base. The effect of these two conditions on the distribution of stresses is similar, although more pronounced in the latter.

A shortening of either the span or radius in the lower part of the dam results in a less deflection of the vertical beam and a greater arch stress. Consequently, the stress line, 16–14, in the lower part of the dam will approach the line, 12–9. For the usual shape of site, the reduction of deflection in the lower part of the dam causes also reduced deflections in the upper part, which part, still having the original radius, suffers a decrease in arch stress, or a movement of the stress line, 18–16, toward the line, 8–12. The net result, therefore, of a shortening of either the radius or span, in the lower elevations, is a reduction in the effect of vertical beam action at all elevations.

Thus far, the conditions considered permit of approximate mathematical determination of the arch stresses, distribution of arch and vertical beam loading, and the deflection of the dam. Such calculations, however, are long and intricate; but, as mentioned heretofore, they have been made, and the results used in the design of arch dams.

A theoretical dam, designed as a pure arch, using Eq. (33), a constant arch stress, 8–9, and neglecting the effect of vertical beam action, will, if checked by such rigorous investigations, be found to have reduced arch stresses in the lower part and increased stresses in the upper part, as indicated roughly by the curve, 18–16–14. However, the increased stresses in the upper part will approach, but not exceed, double the stress assumed, and are amply compensated for by the factor of safety embodied in the usual working stresses. It is probable, indeed, that, for thin arch dams, such as would result from the use of the customary working stresses in Eq. (33), the vertical beam action will not greatly affect the stresses near the top. Moreover, a practical design embodies an appreciable thickness at the water surface, resulting in a considerable reduction in the stresses in the upper part of the dam.

Rigorous, intricate investigations, for the determination of the arch stresses due to a combination of arch and vertical beam action, therefore, are hardly justified, particularly in consideration of the sometimes compensating and sometimes cumulative effect of stresses due to temperature changes, changes in moisture content, and other indeterminate factors which are sufficient to nullify practically the basic assumptions in such investigations. A glance at the deflection curves of the Barren Jack Creek Dam (Fig. 60), will indicate at once the utter futility of endeavoring to arrive at an exact determination of the effect of vertical beam action.

Another condition which affects materially the results to be obtained from rigorous investigations is the probability of variation of the modulus of elasticity of the masonry at different elevations. It is hardly reasonable to assume that, with the varying materials which must be used in different parts of a concrete structure, the modulus would have a great degree of uniformity, particularly when plums are used only in the lower and thicker part of the dam.

A considerable reduction in temperature results in an opening of vertical building joints which must be closed by deflection under water pressure before arch action takes place. This has the effect of increasing the load carried by the vertical beam and reducing the load carried by the arch. An increase in temperature will have the reverse effect.

Swelling of the concrete, due to saturation as the water rises in the reservoir, will have the same effect on the distribution of the loading as a rise in temperature. It is probable that such swelling is nearly compensated by the previous shrinkage of the concrete due to the setting of the cement.

54. Vertical Beam Stresses. As pointed out heretofore, a close determination of the stresses in the vertical beam is difficult if not impossible if all influencing conditions are considered. Fortunately, such stresses are not important, except in very high dams.

Near the base, the effect of the water load is to increase the vertical compressive stresses near the down-stream face. Concrete, in common with other materials, expands laterally when subjected to vertical compression. Being confined between the walls of the canyon, the direct result is an initial negative arch

deflection in an up-stream direction which must be eliminated by water pressure before the vertical beam can deflect down stream and the vertical compression be increased at the toe. Principally to this is attributed the fact that in no existing arch dams has the deflection of the vertical beam resulted in a rupture of the masonry due to excessive vertical compressive stresses, although several arch dams more than 200 ft. in height have been constructed.

The question of horizontal shear in the vertical beam, though equally indeterminate, has no influence on the ultimate strength of the structure, as a failure in this respect, though objectionable as affecting leakage, would result in a better distribution of the arch loading.

55. Recommendations for Design. It has been mentioned that the influence of vertical beam action is to increase the arch stresses in the top of the dam, but that the ratio of the resultant stress to the arch stress, computed from Eq. (33), cannot theoretically exceed two. In view of the fact that the upper part of the dam, which theoretically can reach zero thickness at the water surface, is for practical reasons considerably in excess of the computed thickness, such increase in the calculated stress is inappreciable in comparison with the factor of safety embodied in the usual working compressive arch stresses.

It is recommended, therefore, that the influence of beam action be neglected, and that the thickness of the dam be determined from Eq. (33). The effect of temperature changes in combination with other influences is indeterminate, and, therefore, must be also compensated for by the factor of safety in the unit stresses.

A discussion of working stresses is given in Art. 58. Fig. 51, plotted from Table XXIV, indicates the relation between maximum arch stresses and the height of existing structures. The arch stresses were calculated from Eq. (33). It is significant that the arch stresses are much lower for the higher dams. This may be attributed to the adoption of a larger margin of safety in the higher structures, and also to the desire for an excess of thickness, near the base, on account of the greater vertical pressures.

On account of the uncertainty as to the actual stresses in arch dams, the design should be carefully compared with similar existing structures, and due allowance made for any variation in shape and local conditions which may make for increased stresses. Until our knowledge of the subject has considerably increased, arch

dams differing materially from those already built must be considered in the nature of experiments.

Although dams of the arch type have proved their reliability, on account of the great number now in successful use, without a

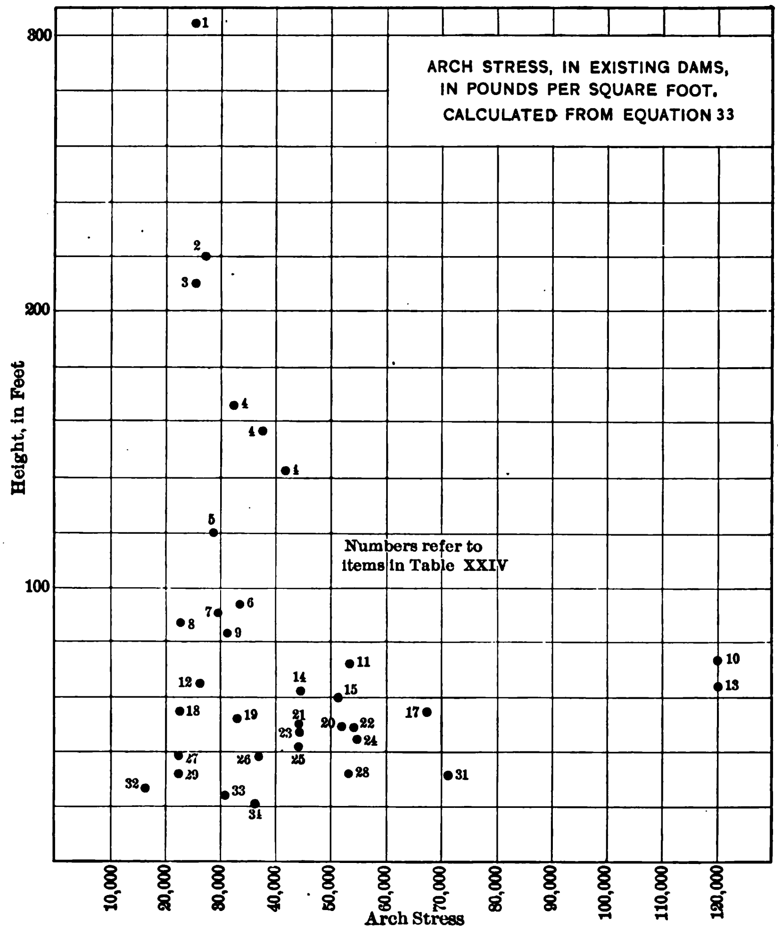


FIG. 51.

single failure, such structures can hardly be expected to be received at present with equanimity above communities of large size, on account of their *apparent* stability being so much less than that of the well-known gravity types. Succeeding years will probably

bring increased confidence in this perfectly safe and most economical type of masonry dams.

56. Details. The outlines of most of the existing arch dams have not been proportioned according to any standard practice, thus resulting in many types. Most of them have a vertical upstream face and a constant radius. Mr. Lars R. Jorgensen * has recently shown that, theoretically, the greatest economy of material in an arch is obtained when it is subtended by a central angle, δ, between abutments, of 133° 34′ (see Fig. 52.)

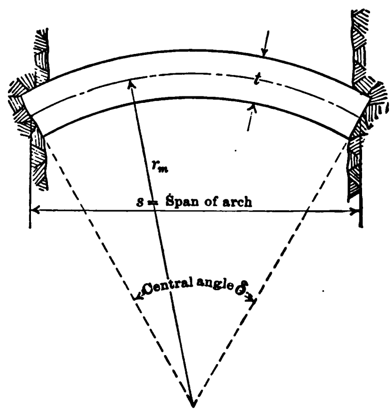


FIG. 52.

This can be proved as follows:

At any elevation, the area, A, of any arch slice between abutments is,

$$A = \frac{2\pi r_m t \delta}{360^\circ} = k' r_m t \delta,$$

From Eq. (33),

$$t = \frac{q r_u}{p} = \frac{q\left(r_m + \frac{t}{2}\right)}{p}.$$

From which,

$$t = \frac{q r_m}{p - \frac{q}{2}}.$$

* *Transactions*, Am. Soc. C. E., Vol. LXXVIII, p. 685.

As p is the constant working stress, and q is a constant at any given elevation,

$$t = k'' r_m.$$

Also,

$$r_m = \frac{s}{2 \sin \frac{\delta}{2}}, \quad \text{and } s \text{ is a constant.}$$

Substituting these values of r_m and t in the first equation. there results,

$$A = \frac{k\delta}{\sin^2 \frac{\delta}{2}}.$$

Differentiating this expression with respect to δ, and equating the differential coefficient to zero, there results for a minimum value of A,

$$\delta = 133° \ 34'.$$

If the cost of form work and similar items were included in the derivation of the most economical central angle, its value would probably not exceed 120 degrees. As a matter of fact, the peculiar configuaration of the site usually limits the value of the maximum central angle which can be adopted economically, it often being necessary to follow certain well-defined ridges in order to increase the average elevation of the foundations. In some instances, where the canyon narrows rapidly toward the bottom, a strict application of a constant angle would result in less thickness near the bottom of the dam than at higher elevations, which is impracticable. Consequently, in most cases, the economical proportions can be found only by trial, adopting successively different values of central angles at various elevations; always being guided by the fact that, unless affected by peculiar conditions, a value of about 120° will be most economical. This method will prove tedious, but will be justified in view of the saving in cost which may be obtained.

It will be noted that, for a constant central angle in a V-shaped canyon, the arch radius must decrease toward the bottom of the dam.

It has been shown already that, for spillway dams having large flood discharges per linear foot of crest, it is not good practice to

allow the falling sheet of water to leave the face of the masonry. For such cases the constant central angle theory will not apply, as it usually results in a practically vertical down-stream face (Fig. 56). For large discharges the down-stream face of an arch dam may be shaped to fit the sheet of falling water for its entire height; then, providing a vertical up-stream face, a varying radius may be adopted to keep the arch stress within the limits desired.

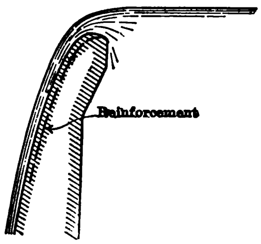


FIG. 53.

In order to reduce the throw of the sheet of water relative to the up-stream face of the dam, the lip of the crest may be made to overhang, as indicated in Fig. 53.

The thickness of the top of the dam, of course, should be proportioned to resist ice pressure, if assumed to exist. The ice pressure may be considered as taken by a portion of the arch equal in height to at least twice the thickness of the arch at the elevation of the ice thrust plus the thickness of the ice, and usually considerably more, depending on existing conditions, the quantity of vertical reinforcement, if used, and the location of the nearest horizontal building joint. The theory is identical with that of the distribution of floor-slab concentrations. It should be noted that, in very few of the dams listed in Table XXIV, was it necessary to provide for ice thrust.

Steel reinforcement has been used to some extent in arch dams, but in most cases has been insufficient in quantity to affect materially the stiffness or the distribution of stresses. The use of continuous horizontal reinforcement is not to be recommended, as it is impractical to anchor the rods thoroughly into the rock of the abutments, and there would be a tendency to draw the dam away from the abutments due to temperature changes when the reservoir is empty. In this respect, the conditions in an arch dam are somewhat different from those in a reinforced arch bridge, as the latter is never without load. Horizontal reinforcement, if used, should not extend through the vertical building joints of the structure, such joints being placed at intervals in radial planes to localize cracks and prevent their formation at too great an inclination to the line of arch thrust.

Vertical reinforcement may be useful to distribute concen-

trated loads, such as ice thrust, and will serve also to stiffen the relatively thin upper part of the dam if the length is considerable in comparison with the thickness.

An arch dam is in reality a long column receiving lateral support only through its connection with the base. It will be noticed that the ratio of curved length to thickness ("ratio of slenderness") at the top of most of the existing dams, is greater than usually allowed in long concrete columns. This is justified on account of the lateral support which the top of the dam receives from the relatively thicker lower portions, and the fact that the arch

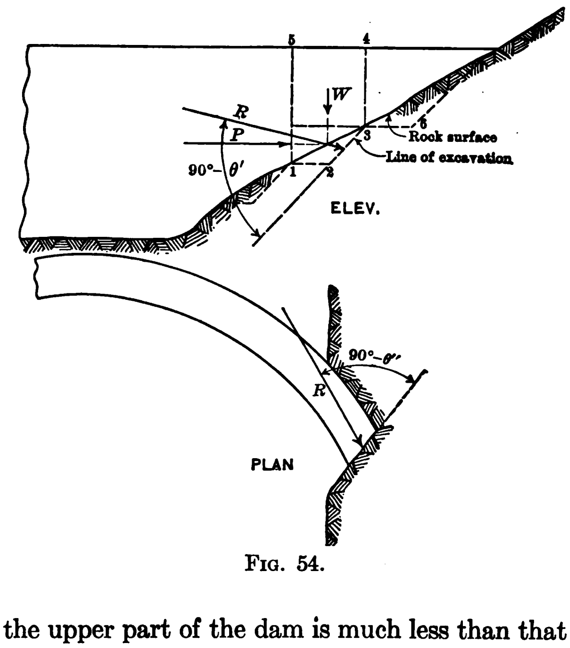


FIG. 54.

stress in the upper part of the dam is much less than that adopted for other parts of the structure.

It is good practice to provide a ratio of slenderness, at mid-height, not greater than 25 and, at the top of the dam, a ratio not greater than 75. The ratio at the top may be somewhat increased if the thickness increases rapidly toward the lower elevations, and the ratio at mid-height is proportionally reduced. This is particularly true if considerable vertical reinforcement is used. In the Salmon Creek Dam (Fig. 56), the ratio at the top is more than 100, but this fact is compensated for by ample

thickness at lower elevations, which, at mid-height is less than 10.

In order to prevent sliding, the rock surface at the abutments should be excavated to the proper inclination to the line of thrust. In Fig. 54, let W represent the total weight of masonry and other vertical forces between the vertical, radial planes, 5–1 and 4–3. Let P represent the total thrust of the arch slice between the two horizontal planes passing through the points, 1 and 3. The rock should be excavated to such lines that the resultant, R, of the forces, W and P, will have a maximum inclination, θ, with a normal to the finished rock surface not greater than the angle of repose of masonry on rock. The angles, θ' and θ'', represent the vertical and horizontal projections, respectively, of the angle, θ. The principle is identical with that of Art. 25 providing for the resistance of gravity dams to sliding.

The number of steps and the depth of excavation to be provided will depend on the character of the rock, and particularly on the inclination of the stratifications. Probable shearing of the rock on a direct line between points, 2 and 6, should be guarded against.

57. Multiple Arch Dams. The theory of arch dams is applicable only in a general way to the arched decks of hollow multiple

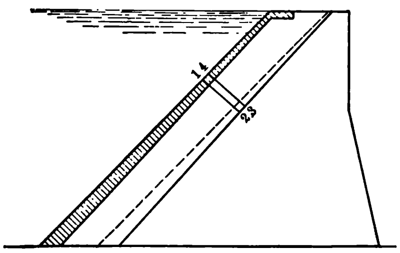


FIG. 55.

arch dams, described in Art. 49. In this type the loading on any arch slice is not uniform, and Eq. (33) does not apply. In Fig. 55, it is seen that, for the arch slice, 1–2–3–4, the unit water pressure at the haunch is greater than at the crown. The percentage in variation of the loading gradually decreases, of course, toward the base of the dam; but, in the upper part of the structure, the non-uniformity of the loading is pronounced.

Therefore the arches in the upper part of the dam should be circular in *horizontal* planes, or well reinforced. In other words, the decks of such dams should be designed as arches subjected to non-uniform loads, the theory of which may be found in many treatises on the subject, such as "Principles of Reinforced Concrete Construction" by Turneaure and Maurer.

58. Allowed Stresses. For the larger and thicker arch dams, 1 : 3 : 6 cyclopean concrete has been commonly used. In the thinner structures the mixture has been 1 : $2\frac{1}{2}$: 5, and even 1 : $2\frac{1}{4}$: $4\frac{1}{2}$, and in the very thin arch decks of hollow gravity dams 1 : 2 : 4 concrete, without plums, is customary.

The stresses in a number of existing arch dams have been calculated from Eq. (33), and are indicated in Table XXIV and Fig. 51. It is seen that, in four cases, a stress of 56,000 lb. per sq. ft. is exceeded, and in two cases the stresses amount to about 120,000 lb. per sq. ft.

These excessive stresses indicate, in a limited way, the factor of safety embodied in the more conservative designs; but they are not in any sense indicative of the stress which should be adopted, for, though the dams have not failed, there is no indication of how close to their ultimate strength they are stressed.

Data on the ultimate strength of masonry have been given in Art. 26. Although the total water load to be carried can be determined quite accurately, there is much uncertainty in the determination of the induced stresses in the dam, due to such loading in combination with temperature changes and other indeterminate factors hereinbefore described. For this reason the stress, calculated from Eq. (33), should not exceed from one-eighth to one-twelfth of the ultimate strength of the masonry, depending on the height and importance of the structure, and the probability of destruction of other property and human life in case of failure.

In very high dams the arch stresses near the base are reduced considerably in order to provide additional width to distribute the weight of the structure.

59. Examples of Arch Dams. Fig. 56 shows details of the arch dam of the Alaska Gastineau Mining Co., on Salmon Creek, Alaska. This dam indicates in a general way the application of the constant central angle theory discussed in Art. 56, the radius gradually diminishing toward the base. To have kept the central

angle constant at all elevations would have resulted in the structure overhanging in places. The 10-ft. triangular piece at the toe was added, in order to distribute more effectually the vertical loads due to the great height of the structure.

Fig. 57 indicates the conservative design of the North Crow Dam for the water-supply system of Cheyenne, Wyo. The computed arch stress is only 23,000 lb. per sq. ft., and the ratio of slenderness is only 11 and 35, at mid-height and top, respectively. A 1 : $2\frac{1}{4}$: $4\frac{1}{2}$ concrete mixture was used throughout the dam, and steel reinforcement was placed in the upper 40 ft., as indi-

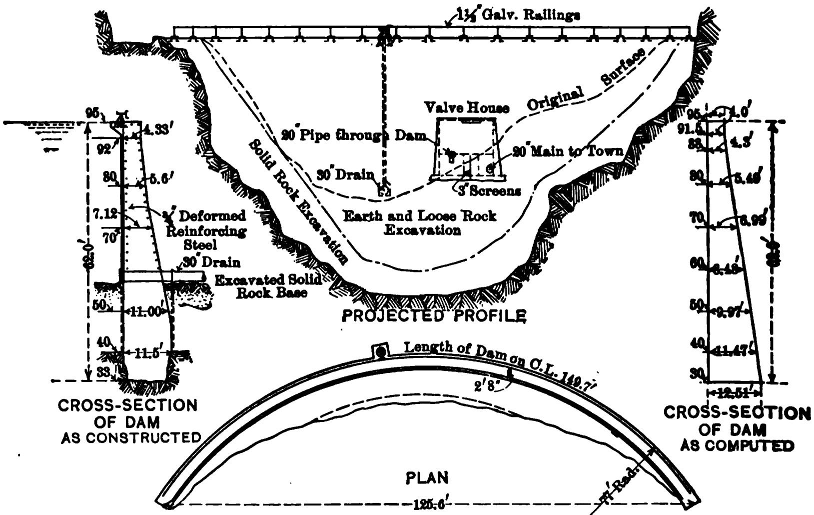


FIG. 57.—North Crow Arch Dam. (*Eng. Record*, Vol. LXVII, p. 149.)

cated. A spillway was provided through a channel excavated in the rock at one end of the dam.

Fig. 59 is a typical example of thirteen arch dams built by the Public Works Department, New South Wales. These dams are all described in Table XXIV, but they do not include the Barren Jack Creek Dam. In all these dams the concrete was mixed in the following proportions:

$4\frac{1}{2}$ parts of Portland cement,
$11\frac{1}{2}$ parts of sand,
10 parts of "shivers" of $\frac{3}{4}$-in. to $\frac{1}{8}$-in. gauge,
13 parts of "metal" of $1\frac{1}{2}$-in. gauge.

FIG. 58.—View of North Crow Arch Dam.

Except in Lithgow No. 2 Dam, the concrete was cyclopean, containing a maximum number of two-man plums. Many of these

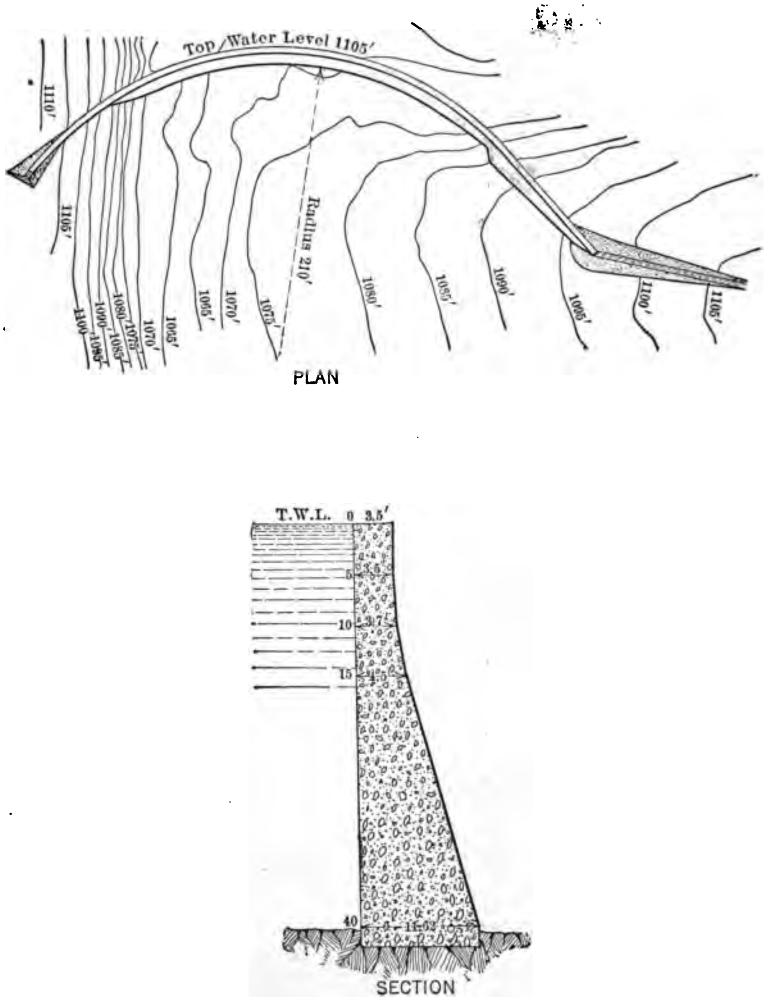
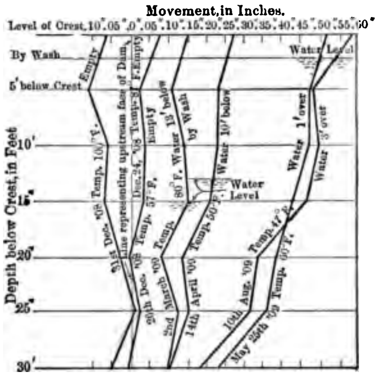


FIG. 59.—Wollongong Arch Dam, New South Wales. (*Eng. News*, Vol. LXIII, pp. 588–589.)

dams are noted for their slenderness, which greatly exceeds that of usual American practice.

In Fig. 60 is indicated the result of measurements of the deflection of the Barren Jack Creek Dam under variations in pond level and temperature of the air. It is probable that the temperature of the water has more influence on deflection than that of



Lines Show Movements of Dam.

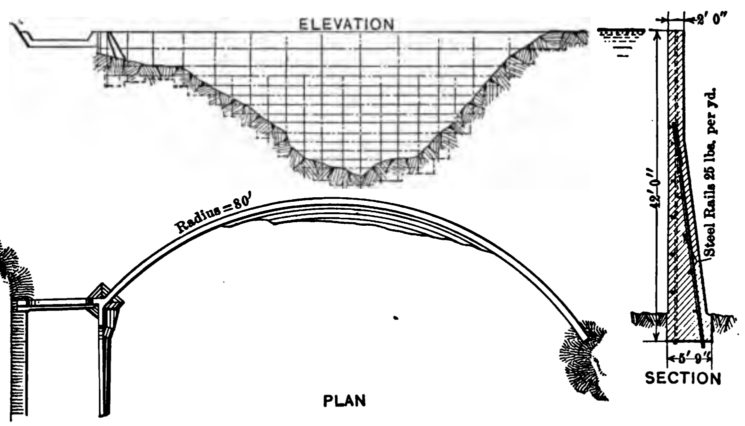


FIG. 60.—Barren Jack Creek Dam, Australia. (*Eng. Rec.*, Vol. LXI, p. 664.)

the air, and it is unfortunate that it was not also recorded. The deflection is seen to be much less than would be expected for a structure of this type.

Fig. 61 shows a plan and section of the arch dam of the Agua Pura Co. of Los Vegas, N. Mex. The dam is quite rigid, the ratio

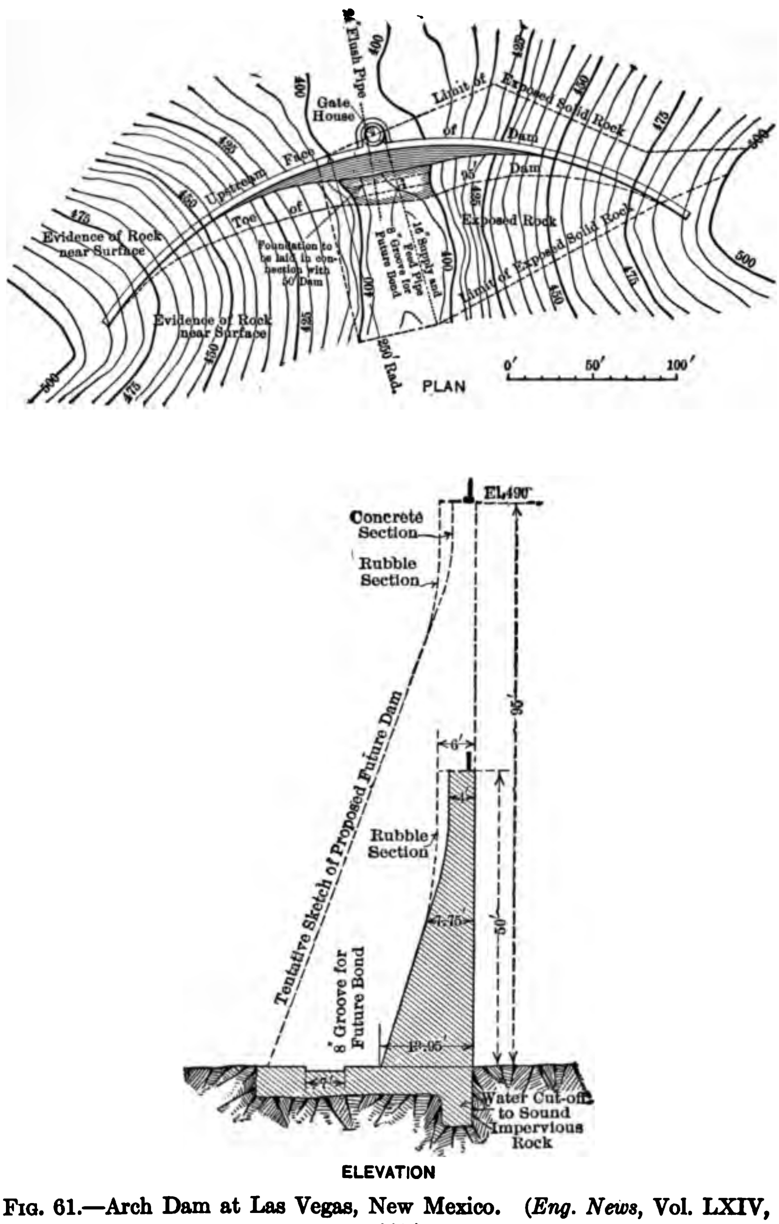


FIG. 61.—Arch Dam at Las Vegas, New Mexico. (*Eng. News*, Vol. LXIV, p. 446.)

of slenderness at mid-height and top being only 17 and 52, respectively. In this dam the radius might have been reduced considerably with a reduction in quantity of masonry for the same arch stress, the central angle being, for the proposed extension, only 88° at the top of the dam and much less at lower elevations. For the existing structure, the central angle at the top is only 48°

CHAPTER X

PREPARATION AND PROTECTION OF THE FOUNDATION

60. General Considerations. A good foundation is of ample strength to withstand the weight of the structure, sufficiently rough to provide ample margin against a sliding failure, tight enough to prevent excessive leakage and uplift, and as clean as possible, if rock, to insure the maximum effectiveness of the bond * between it and the dam.

Absolute tightness is difficult, if not impossible, to obtain. It is possible, with first-class rock, to provide against any appreciable leakage or uplift; but, in the design of dams, the engineer has to contend with foundations varying from solid rock, through all grades of stratified and ruptured rock, to alluvial deposits of a very porous nature.

Considerable preparation is always necessary in order to provide the requisites of a good foundation. It is probable that more than 90 per cent of all failures of masonry dams has been caused by faulty foundations. It is of the utmost importance, therefore, that this feature of the design should receive the proper amount of attention. It is unfortunate that the designer is not always the builder, as many of the assumptions used in the design will depend on the extent and character of the treatment which the foundation receives. In all cases the designer should prepare the specifications for the construction of the dam, and preferably have supervision over the work.

61. Rock Foundations. It is not within the province of this book to enter into a discussion of construction problems in connection with the proper preparation of rock foundations. For this, the reader is referred to C. W. Smith's "Construction of Masonry Dams"† and works of like character. There is pointed out, however, in addition to the necessary designing features, the

* Bond between rock foundations and the dam, although neglected on account of its unreliability, always exists to a certain extent, and undoubtedly adds considerably to the stability of the structure.

† McGraw-Hill Book Co., Inc., 1915.

results which should be obtained from such preparation, in order to justify the adoption of the designing assumptions herein recommended.

Surface rock is usually badly weathered, and unsuitable for the support of a dam. It is sometimes necessary to excavate to considerable depths before rock of an acceptable nature is uncovered.

In the excavation of rock foundations, it is always necessary to take particular care in order that good rock directly beneath the blasting charges is not unnecessarily shattered. It is often specified that the last foot or two of the excavation shall be barred and wedged loose. The proper method will suggest itself to the experienced builder when it is borne in mind that no part of the final foundation should be disturbed from its original position and that no stratifications should be jarred loose.

It has been pointed out heretofore that there should be as much resistance to sliding below the surface of the foundation as at other planes. If, therefore, the foundation contains loose horizontal or nearly horizontal stratifications on which there is danger of sliding, the excavation should be deep enough to obtain a " toe hold " on the rock below the dam, in order that some, if not all, of the horizontal loading may be carried to the rock by direct compression through the vertical plane at the toe between the dam and the rock.

In order that the masonry of the dam shall have the maximum possible adhesion to the foundation, it is necessary that the final rock surface should be absolutely clean, and the presence of flowing water should be rigidly guarded against. There is nothing better for cleaning rock surfaces than jets of clear water, under considerable pressure, and the thorough use of stiff wire brooms. The jet is particularly adaptable to cleaning out vertical seams and pot-holes of considerable depth. Such seams should be well plugged with mortar. For concrete dams it is often specified that the finished foundation shall be covered with a thin coat of rich mortar immediately before the concrete is poured. Springs in the foundation are not usually plugged before the masonry is started, the practice being to allow them to discharge through pipes until a sufficient mass of masonry has been placed to balance any possible uplift from them. The springs are then grouted under pressure.

It is essential that the plane of contact between the foundation and the masonry should be as rough as possible, in order to assist in the resistance of the dam to sliding. In the case of clearly defined horizontal stratifications, it is not always possible to obtain a rough surface, and a toe hold on the rock below the dam must be provided, for reasons explained above.

Some leakage or seepage through the foundation is to be expected. Aside from the sometimes serious waste of water, leakage is objectionable because it provides a possible means of uplift and, in certain classes of foundations, scour. Foundations, in general, have a tendency to tighten gradually, particularly if silt is present in the water. The exception is in certain classes of limestone foundations, where the water has a solvent action on the rock, and where there is not sufficient silt in the water to plug the leaks thus formed. It must not be supposed that, because the stream in its original state carried large quantities of silt past the dam site, this condition will obtain after the dam is constructed. In large storage reservoirs silt will not reach the dam, even during the greatest floods, until perhaps after a great number of years, when the whole volume of the reservoir has become silted up and its usefulness destroyed.

In order to confine the leakage to a reasonable quantity, it is necessary, with poor foundations, to provide a cut-off or artificial impervious barrier under the heel of the dam. For rock foundations there are two general types of cut-offs; first, a trench filled with concrete, and second, holes drilled at frequent intervals and grouted under pressure.

The first type is much to be preferred if it can be constructed at reasonable cost. Before the use of grouted cut-offs was common, the first type was sometimes carried, in exceptional cases, to depths of 50 ft. or more. Its advantages are twofold, in that it not only provides perfect inspection of the vertical area to be improved, but is a more tangible and sure method of obtaining the desired end. For the excavation of the trench, even more care must be taken not to shatter the surrounding rock, particularly at the bottom. For the cut-off trenches of a number of important dams, where the excavation has been difficult, holes have been drilled a short distance apart to form planes of weakness to which the sides of the trench break without serious disturbance outside of the limits desired. The excavation has sometimes been made

with a channeling machine, but this method, of course, is very expensive and hardly justified for any case, unless the rock is easily cut. The concrete for the cut-off, in rock foundations, is usually built under the same specifications as for the rest of the dam, and provided with the same system of joints.

Cut-offs of the second type have been more common of late years. Typical examples may be found at the Estacada Dam, in Oregon; * the Lahontan Dam, of the Truckee-Carson project, U. S. Reclamation Service; † the Horseshoe Falls Dam on Bow River; ‡ and the Arrowrock Dam, of the U. S. Reclamation Service.*

A method of grouting which seems to have given the best results consists in drilling a primary series of holes in a row under the heel of the dam, from 10 to 15 ft. apart on centers, and of a depth depending on the nature of the foundation and the head of water to be sustained. The holes are usually about 3 in. in diameter.

The primary holes are first subjected to water pressure, preferably from a tank placed at the same height as the crest of the dam, or a little above it, and the rate of leakage from each hole recorded. The flow of water also serves to clean out the earth seams in the vicinity of the holes in preparation for the grout which is to follow.

After the primary series of holes is grouted, a second series consisting of an equal number of intermediate holes in the same row is then drilled, tested, and grouted. If necessary, a third series is drilled and treated, thus reducing the spacing to a quarter of that for the first series. The result of a test of any hole is considered an indication of the relative tightness of the foundation between the two adjacent holes previously grouted. The process, therefore, is continued until the tests indicate that the leakage has been reduced to a satisfactory extent.

The upper ends of all holes to be grouted should be provided with threaded pipes with which to make connection to the grouting machine. These pipes must be anchored or weighted to prevent a blow-out during the process of grouting. This is sometimes

* For description see paper by Harold A. Rands, *Transactions*, Am. Soc. C. E., Vol. LXXVIII, p. 447.

† For description see article by D. W. Cole, *Engineering News*, Apr. 3, 1913.

‡ For description see article by H. S. Johnson, *Engineering Record*, Dec. 12, 1914.

done by cementing the connecting pipes into the drilled holes, or placing them in a concrete cut-off carried far enough into the rock to provide ample grip. The drilling, in the latter case, is usually done through the pipes. When it is desired that the drilling shall not interfere with the erection of masonry, the pipes may be carried up along with the masonry and the operations of drilling, testing, and grouting conducted from whatever elevation the masonry has reached.

If it is apparent from the borings that there is an open seam close to the surface, care should be taken that there is sufficient weight above the seam to balance the grouting pressure. Part of the dam may be previously built to assist in this respect.

The initial grouting pressure, for each hole, should be that which is necessary to force the grout slowly into the hole. The pressure is raised gradually, as the hole tightens, so as to disturb the natural formation as little as possible.

The grout usually consists of a mixture of neat cement and water, of proportions varying to suit the character of the foundation. For porous rock, with fine seams, a mixture as thin as 1 of cement to 8 or 10 of water has been found satisfactory. Where the seams are large, and other voids exist, a thicker mixture must be used, gradually changing to a thinner mixture as the hole tightens to refusal.

The grouting operations are usually started from one or two ends of the site, the holes being treated successively. Each hole should be capped, if necessary, as soon as it has been grouted, in order that the grout to be forced into the next hole will not flow back through the completed hole and be wasted, instead of passing on to the ungrouted portion of the foundation.

If grout issues freely from an untreated hole, indicating an open seam between it and the hole being grouted, the untreated hole may be capped and the grouting operation considered as serving the two holes. The object is to supply grout to each hole in sufficient quantities and at the desired pressure, and it is immaterial whether it is supplied at the top of the hole or by way of an open seam from another hole. Should the grout from an untreated hole issue sluggishly, indicating an indirect or only partly free connection, the hole should be capped, but should receive its share of treatment in due course.

When thick grout is being used, care must be taken that grout

from one hole does not partly fill an adjoining hole and set before the latter can be treated. When thick, grout sets much more quickly than when thin. It is sometimes advisable to provide sufficient shifts of men to conduct the grouting operations continuously.

All holes should be gone over the second time after the grout has set. If the grout is thin, considerable settlement in the hole will be observed, and it is often possible to inject an additional quantity.

No one method of grouting will apply for more than one site; in fact it is usually found advisable, for each case, to change the adopted method several times during a course of treatment, due to experience constantly being gained as the work progresses.

The following is an abstract from the previously mentioned article on the grouting of the Lahontan foundation. The dam is an earth embankment 124 ft. high with a core-wall, but the process of grouting would apply equally well to a masonry dam.

The grouted portion of the foundation consists principally of a formation resembling red sandstone, varying from solid rock to a tough red clay or even an unstable sandy clay, containing many intermixtures and stratifications.

In the concrete cut-off trench (Fig. 62*a*), which was 30 ft. deep, were previously set 5-in. well casings in two rows 2 ft. apart. The casings were 3 ft. apart in each row, and were staggered, so that there was a casing for every 18 in. of length of cut-off, as indicated in Fig. 62. Core drillings, with $2\frac{3}{8}$ in. outside diameter bits, were made through the well casings and extended at least 30 ft. below the bottom of the concrete cut-off. Some of the holes were carried 70 ft., and the average was 40 ft., below the trench, or 70 ft. below the river bottom.

The work was divided into two sections, on account of the necessity of reserving half the width of the site for the diversion of the stream.

In the first section, alternate holes, 6 ft. apart in the upper row, were drilled, tested, and grouted; the intention being to complete the upper row, and then the lower row, in successive steps. Those of the first series were designated " primary holes." Testing was done under a head of 127 ft. The rate of flow through each hole was recorded.

After the holes of the first series were grouted, the intermediate,

or "secondary holes," in the upper row, were drilled, tested, and grouted, thus completing the grouting in the upper row. Fig. 62 indicates the results of all tests, and the number of bags of cement used in each hole. It will be noticed that the grouting of

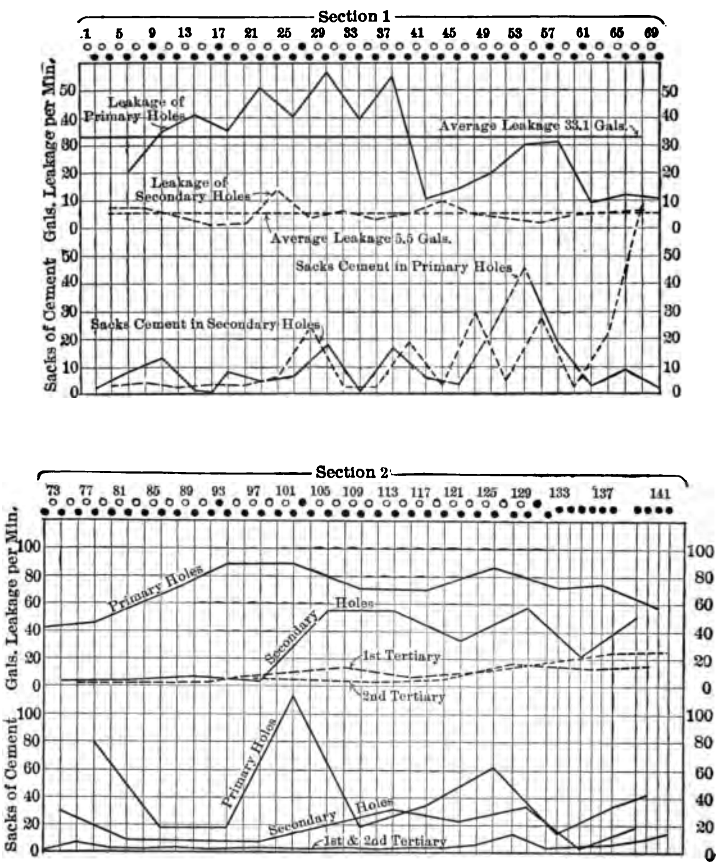


FIG. 62.—Gallons of Leakage Observed and Sacks of Cement Necessary to Grout Different Borings in Lahontan Dam Foundations. (*Eng. Record*, Vol. LXVII, p. 340.)

the "primary holes" reduced the average leakage per hole from 33.1 to 5.5 gal. per min. It was anticipated that the leakage would be further reduced by the grouting of the "secondary holes," so that very few of the holes in the lower row were drilled. It is

claimed that the quantity of cement which the holes took was not indicative of their relative tightness, on account of caving of the holes resulting in varying volumes to be filled, and the escape of grout into adjoining well casings.

The results obtained for the first section of the dam led to an alteration in the procedure for the second section. In the latter the holes were drilled, tested and grouted in the following order:

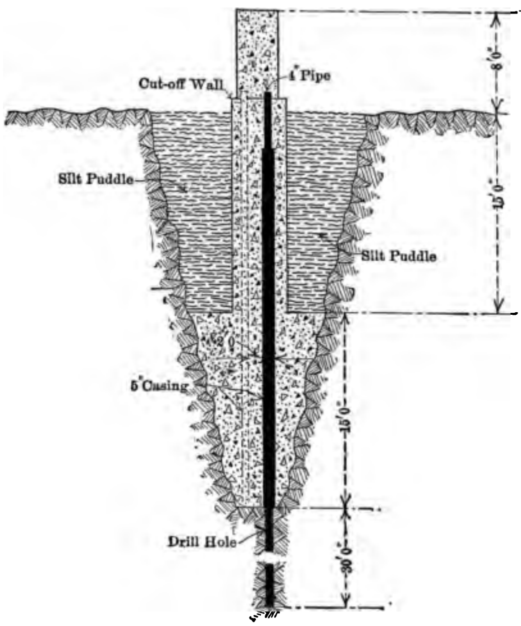


FIG. 62a.—Grout Pipes in Cut-off Trench of Lahontan Dam. (*Eng. Record,* Vol. LXVII, p. 340.)

" Primary holes " Nos. 78, 86, 94, etc., 12 ft. centers throughout; " Secondary holes," Nos. 74, 82, 90, etc., 12 ft. centers throughout; "1st Tertiary holes," Nos. 76, 84, 92, etc., 12-ft. centers throughout; "2d Tertiary holes," Nos. 80, 88, 96, etc., 12-ft. centers throughout.

The average leakage per hole for the second tertiary holes was 6.4 gal. per min., indicating the necessity of grouting very few holes in the second row.

All grouting was done with a duplex-cylinder, air-stirring, Caniff

grouting machine; a pressure of 25 lb. being used at the beginning of each operation, increasing gradually until finally the grout was driven home at a pressure of 100 lb. per sq. in. A mixture of 1 part of cement to 7 or 8 parts of water was found most desirable, although the mixture was thickened considerably when the flow appeared to be too free.

Uplift pressure from head-water cannot be eliminated entirely unless the cut-off is absolutely tight. With a deep, well-built, masonry cut-off or a grouted cut-off, constructed under favorable conditions, the uplift may be negligible.

Where it is thought that the effectiveness of the cut-off is not thoroughly reliable, and in other cases where unusual precautions are desirable, it is the practice to provide drains in the foundation to facilitate the escape of whatever water finds its way past the cut-off.

For this purpose holes may be drilled at intervals across the site in a row just below the cut-off and provided with a free connection to tail-water. The holes should be drilled to or a little above the elevation of the bottom of the cut-off. For grouted foundations the holes should be drilled after all grouting operations have ceased.

The holes are usually 10 or 15 ft. apart; but the spacing depends greatly on local conditions. Other things being the same, the proper spacing should vary directly as the depth of the holes, and directly as the perviousness of the foundation relative to that of the cut-off.

The holes also serve the purpose of indicating the extent of leakage past the cut-off, and, for this reason, should discharge a little above tail-water, in order that the quantity of flow may be observed.

Box drains, open-jointed pipe, and other types of drains, used without drilled holes, have often been placed between rock foundations and the dam, and may be effective in eliminating uplift at that elevation; but they are not adaptable to rock foundations containing nearly horizontal open seams near the surface, as they have no direct connection with such seams, and may be separated therefrom by a horizontal layer of very impervious rock.

For the usual spillway section, the smooth thin sheet of swiftly moving water issuing horizontally from the bucket at the toe of the dam, may, under certain conditions, retain a uniform depth

and velocity until, at a point, perhaps, considerably remote from the dam, it will suddenly increase in depth to tail-water level, forming a standing wave, as indicated in Fig. 4, and as described in the latter part of Art. 14. Under such conditions the river bed below the dam will be subjected to a velocity much higher than normal. The impact from swiftly moving water is known to be very destructive to rough, soft, rock surfaces, particularly if there are seams into which the jet may enter. In the latter case the velocity head may be converted into a pressure head capable of lifting large masses of rock.

In all cases, where the rock surface is known to be soft or stratified, an apron of concrete, or other suitable material, should be provided to afford protection to the foundation at the toe of the dam. However, as explained above, there may be danger, under certain conditions, of the water not returning to normal tail-water level until after it has passed the end of the apron. The likelihood of undermining the toe of the apron, in such cases, is evident, and systems of baffles forming a partial obstruction to the flow, have sometimes been built to insure the return of the sheet of water to tail-water level before it has reached the end of the apron.

The concrete baffles used for the Gatun spillway dam are indicated in Figs. 6 and 7. During large flows, the jet, when it reaches the baffles, is about 6 ft. thick, and flows with a velocity of about 60 ft. per sec. The baffles are 9 ft. high. The effect of the baffles, as indicated in Fig. 7, is to increase the depth to tail-water level or about 20 ft. deep, and to reduce the velocity to 18 ft. per sec. The baffles piers are heavily reinforced, and are protected on their up-stream faces with thick iron castings.

Where the profile of a spillway dam site is not horizontal, the water passing the ends may flow along the toe of the dam toward the middle with high velocity and destructive force. Fig. 63 is a view of the down-stream face of the Estacada Dam. It will be noted that training walls are provided to conduct the water to a safe distance.

62. Earth Foundations. Masonry dams on earth * foundations are numerous; but their use has been practically limited to structures not more than 50 ft. high for good hardpan and 30 ft. for less resisting earth. This limitation in height may be attrib-

* Earth, as here used, may be considered as all kinds of material not usually classed as bed-rock.

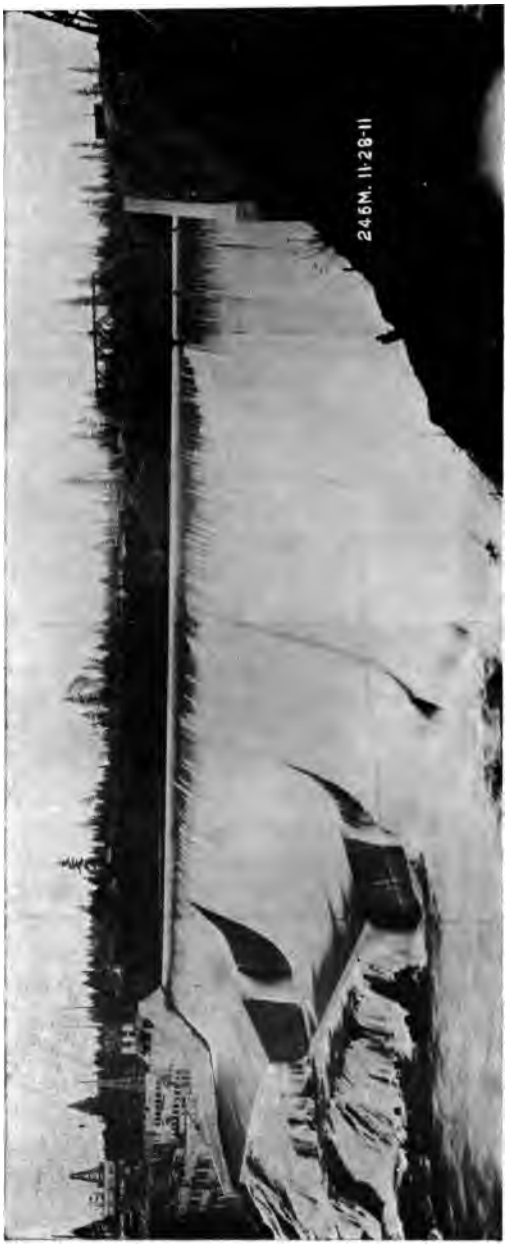


FIG. 63.—Dam at Estacada, Oregon.

uted to the fact that the treatment of earth foundations, to prevent erosion and excessive seepage, requires an expenditure far in excess of that necessary for the foundations of dams on rock. In fact, the cost of foundation treatment for dams on earth is often the major part of the total cost of the structure. Consequently, for moderate and high dams it will be found best to adopt another type of structure, or change the site. There have been few precedents for dams higher than noted above, although structurally there would seem to be no reason for a limit to the height, provided sufficient funds are available to meet the unusual expense.

It is essential that there be no excessive, unequal settlement of the dam, as the tightness of the structure is dependent on the absence of settlement cracks.

The preparation of the foundation for a dam on earth must be made with four objects in view:

a. To prevent excessive seepage under the dam;
b. To prevent scouring by the water passing over the dam;
c. To provide ample bearing strength;
d. To prevent sliding.

Excessive seepage, or underflow, through the foundation is objectionable, not only on account of the waste of impounded water, but principally because of the danger of movement of the particles of the foundation due to its erosive power.

Where practicable, seepage should be prevented by carrying a tight cut-off, under the heel of the dam, to an impermeable formation. The cut-off may consist of a concrete diaphragm or a series of grouted holes, as described for rock foundations; interlocking steel sheet-piling, or tongued and grooved wood sheet-piling. Sheet-piling, unless driven under favorable conditions and with extreme care, is very apt to leak badly; and this is a condition to be avoided, as recent experiments have indicated that very slight leakage is sufficient to destroy its effectiveness. Wooden sheet-piling for cut-offs should never be used where the foundation contains boulders which are large enough to cause the piles to buckle or deflect. Even steel sheet-piling has been made useless under very heavy driving.

Figs. 39 to 42, inclusive, show details of the Mathis Dike Dam, of the Tallulah Falls water-power development. The dam is built on sand mixed with very little clay and decomposed mica

schist. A concrete cut-off wall was provided, extending from 30 to 57 ft. to rock, except at one end where, as indicated in Fig. 39, holes were driven to rock and effectually grouted under a pressure of 70 to 85 lb. after the reservoir was filled. Before grouting, these holes were opened to the lower side of the dam, and were found to leak from 0.5 to 12.0 gal. per min. The cut-off wall is composed of 1 : 3 : 6 : cyclopean concrete, with tongued and grooved vertical contraction joints. The upper 15 to 20 ft. of the wall is reinforced in order to prevent shrinkage cracks. The sheet-piling indicated in the illustration was an experiment, but, owing to the nature of the foundation, was abandoned after 50 ft. had been driven.

Where it is impracticable to carry the cut-off to an impermeable formation, undermining may be prevented by providing a path of enforced percolation of sufficient length. The velocity of the underflow uas been found by experiments to be directly proportional to the hydrostatic head * and inversely proportional to the length of the path of percolation. Its erosive force depends on its velocity. The required length of path of enforced percolation, therefore, is a direct function of the head on the dam, and depends on the nature of the material in the foundation. W. B. Bligh † gives the following empirical formula:

$$l = Ch_n, \qquad \qquad (44)$$

where l = the length of the path of enforced percolation in feet; see Fig. 64, in which $l = l_1 + l_2 + l_3 + l_4 + l_5 + l_6$;

h_n = the net head on the dam, in feet;

and C = a coefficient which depends on the character of the material in the foundation.

For C, Mr. Bligh recommends the following values, which are based on the dimensions of a number of existing dams on earth foundations:

$C = 18$ for mud or silt;

$C = 15$ for fine micaceous sand;

* Experiments by D'Arcy, Hagen, Hazen, and others, indicate that the flow of water through fine sand and gravel is similar to the flow through capillary tubes, the velocity being directly proportional to the first power of the head.

† "Dams, Barrages, and Weirs on Porous Foundations," *Engineering News*, Dec. 29, 1910.

$C = 12$ for coarse-grained sand;

$C = 9$ for a mixture of sand and gravel;

$C = 5$ to 9 for clay, shale, or a mixture of sand, gravel, and boulders.

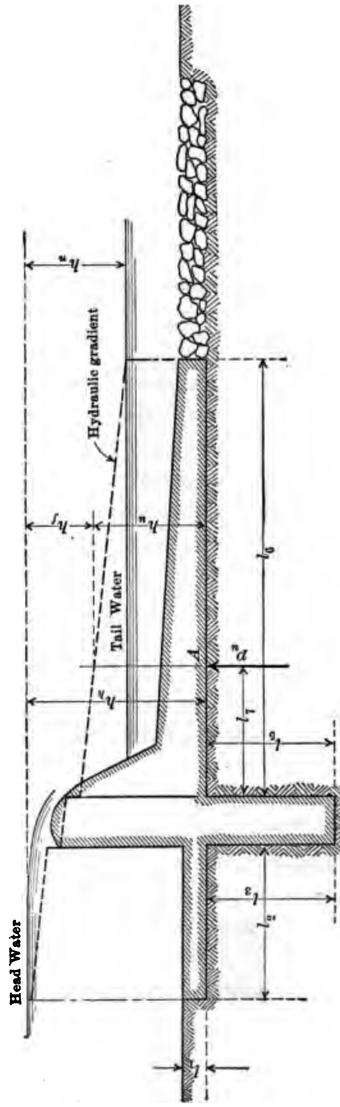


FIG. 64.—Theoretical Hydraulic Gradient for Dam on Earth Foundation.

Care should be exercised in the application of these coefficients, as it is impossible to convey an exact description of the characteristics of the foundations on which they are based. Therefore, ample margin should be allowed, unless careful comparison has been made between the materials in the foundation and those in existing structures of like nature.

It has been proved experimentally that the length of the path of percolation, as affecting uplift pressure and erosive force, is not the shortest distance between head- and tail-water, but the length of the actual plane of contact between the structure and the earth, including all cut-offs, provided the cut-offs are not closer together than twice their depth. In other words, the effective length of the path of percolation, as indicated in Fig. 64, is the summation of the distances, l_1, to l_6, inclusive.

The proper length of the path of enforced percolation may be obtained by providing a down-stream apron, an up-stream apron, one or more cut-offs, or a combination of these parts, as indicated in the accompanying illustrations. Many types and combinations have been proposed and constructed. There seem to be no standards in this respect. The choice will depend, to a large extent, on local conditions. A single masonry cut-off at the heel is commonly provided for good earth foundations; but it is often found advisable to substitute an up-stream or down-stream masonry apron, which is theoretically as efficient as a cut-off of half its length.

The Jamrao and Barra type of weirs, indicated in Figs. 65 and 66, were designed for localities where the foundations are composed of very pervious material, requiring a long path of enforced percolation.

Fig. 67 is a section of a portion of the Granite Reef Dam of the Salt River Project. The dam is founded on a formation of gravel and boulders. The coefficient, C, from Eq. (44), was only 4.2, the length of the path of percolation being measured on the under surface of the concrete from the river bed at the heel to the first drain hole in the apron. When the dam was first used, considerable water passed under the cut-off and out through the drains in the apron. Fortunately, this flow was soon stopped by the large quantity of silt which was carried by the river and deposited at the heel of the dam, forming an increased and sufficient length of path of percolation.

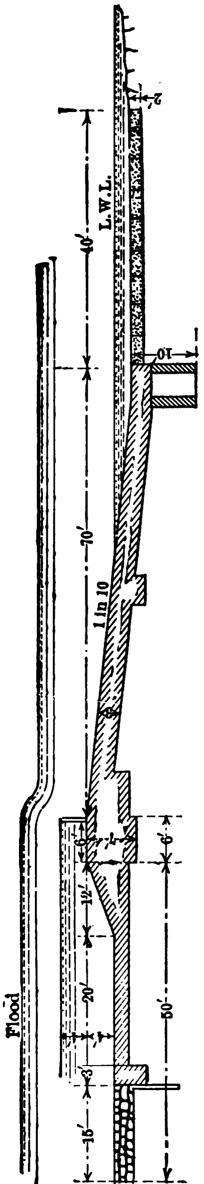


FIG. 65.—Jamrao Weir, India. ("The Practical Design of Irrigation Works," Bligh, p. 174.)

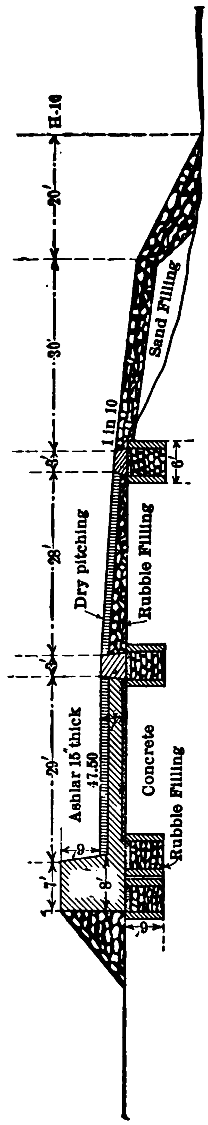


FIG. 66.—Barra Weir, India. ("The Practical Design of Irrigation Works," Bligh, p. 188.)

In all classes of earth foundations, the river bed below the dam must be protected from the wash of the water passing over the crest, in order to prevent undermining the structure, from this cause. The object is to provide, as far as possible, a means for the stream to regain its normal velocity, corresponding to the

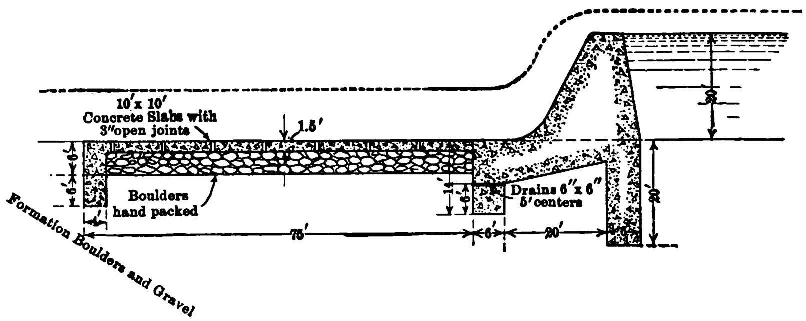


FIG. 67.—Granite Reef Dam, Salt River Project, Arizona. ("The Design and Construction of Dams," Wegmann.)

flow, before the end of such protection is reached. It is common practice to provide a concrete or rock-filled timber crib apron for a short distance below the dam, the end of the apron being protected by an extension of rip-rap, as indicated in the accompanying illustrations. The lower end of the apron is often further pro-

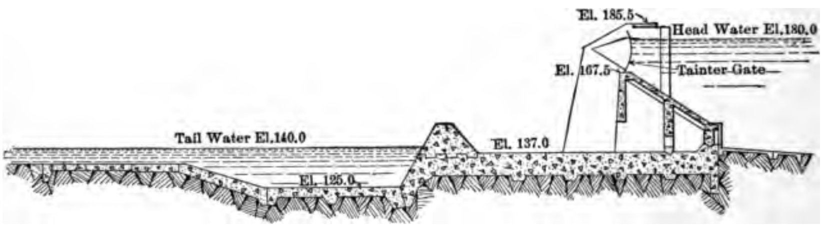


FIG. 68.—Power Dam on Au Sable River. (*Eng. Record*, Vol. LXVI, p. 247.)

tected by a vertical diaphragm of concrete or sheet-piling, which serves to retain the foundation, under the apron, if the rip-rap is washed away. Similar protection has been used at the toe of the dam to provide for a possible failure of the apron.

The usual type of spillway section, passing large flows, is not adaptable to dams on earth foundations unless the tail-water has

sufficient depth to break the force of the falling water,* or unless baffles are provided, as described in the last part of Art. 61.

A type of apron frequently used is indicated in Fig. 68. The water is dropped, in one or more stages (two in this instance), into pools of sufficient depth to destroy the velocity.

In case the bed of the river is higher at the ends than at the middle, the water passing over each end, if not taken care of, will flow parallel to the dam toward the main channel. For steep slopes, this flow may acquire velocities sufficiently great to scour the foundation at the toe of the dam or the end of the apron. This condition should be avoided by the construction of stone fill, masonry, or other suitable training dikes at intervals extending from the dam to a point down stream far enough from the dam to obviate the possibility of damage to that structure. Sometimes the training dikes are supplemented by a system of canals, parallel to the river, in order to provide a gradual descent.† Such construction is obviously expensive, particularly for large flows, and often necessitates the limitation of the length of spillway to the width of the level portion of the river bed.

Proper unit compressive stress on the foundation is usually obtained by spreading the footing of the dam, although supporting piles have been used in some cases. Many types of wood and concrete piles have been used for this purpose. The subject of piles is too lengthy for proper treatment here, and the reader is referred to one of the many books on foundations of that type, such as "Foundations of Bridges and Buildings" by Jacoby and Davis.‡

The weight of the hollow Mathis Dike Dam was distributed over the foundation by a mattress covering the entire base, as indicated in Fig. 41. The weight of solid dams may be distributed through the aprons, which are often reinforced for that purpose.

Where sufficient length of enforced percolation is provided by a cut-off at the heel of the dam, the base is usually drained, to prevent the possibility of uplift; unless, of course, the weight of the dam is great enough to sustain the uplift pressure, as hereafter discussed. The best type of drain for this purpose consists of an inverted filter surrounding open tile pipe, although many other

* See Art. 14.

† See "The Laguna Dam," *Engineering News*, Feb. 9, 1905, and Feb. 27, 1908.

‡ McGraw-Hill Book Co., 1914.

types have been used. The main drain should be placed immediately below the cut-off, at the heel of the dam, as shown in Fig. 70. Drainage is often provided, even though the cut-off extends to impervious material.

Where it is possible to reach impervious material, it is common practice to place the only cut-off at the heel of the dam, although lower rows of auxiliary sheet-piling and even concrete diaphragms are sometimes provided, in order to prevent undermining from water passing over the crest, in view of a possible failure of the apron or apron extension. Examples of such auxiliary piling and diaphragms are indicated in Figs. 70 and 67, respectively. In such cases they should be well perforated or drained, in order to

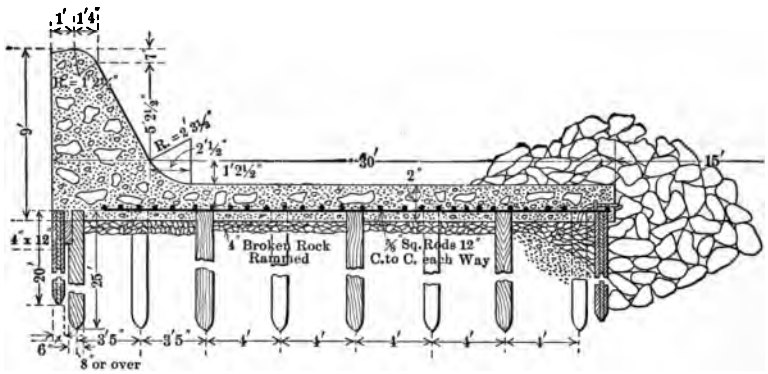


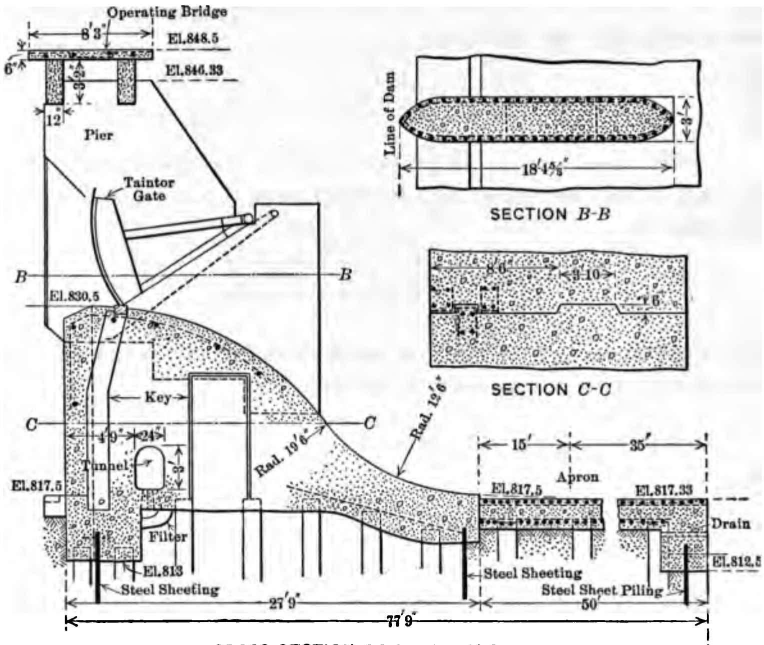
FIG. 69.—Diversion Dam of the Rio Grande Project, New Mexico. ("Irrigation Practice and Engineering," Vol. III, Etcheverry.)

hold back no leakage which may pass the upper cut-off and main drain. Intermediate drains are also desirable, if the area of the base of the dam or apron is large. In brief, to prevent uplift on the base of the dam, the cut-off at the heel should be as tight as possible, and the foundation below the cut-off as pervious as possible.

The Rio Grande Dam was drained by providing a mattress of well-rammed broken rock under the entire base and apron. Leakage is discharged through the pipe over the lower row of sheet-piling indicated in Fig. 69. The mattress of the Mathis Dike Dam was well perforated.

Fig. 70 is a section of the spillway of the Coon Rapids power development on the Mississippi River. This dam is built on a

deep bed of glacial drift, and is carried on a pile foundation. The sheet-piling at the heel was driven to penetrate at least 5 ft. into impervious material, which varied from 2 to 25 ft. below the surface. The filter, drain, and tunnel are intended to provide for the escape of any flow which may pass the heel, thus relieving the dam from upward pressure. A similar row of sheet-piling was



CROSS-SECTION OF SPILLWAY SECTION

FIG. 70.—Coon Rapids Dam on the Mississippi River. (*Eng. News*, Vol. LXXVII, p. 118.)

driven at the toe of the dam, which, in the author's opinion, should have been perforated or drained.

The sheet-piling at the end of the apron was carried down 8 ft., is intended to prevent the scour from undermining the apron, and is drained as indicated.

Should the base of the dam or the apron form part of the required path of enforced percolation, as indicated in Figs. 64 to 67, inclusive, drainage, of course, may not be allowed, and uplift pressure must be provided for.

The pressure head of the underflow, for earth foundations, is usually considered to vary uniformly, along the path of percolation, from full head-water pressure at the up-stream end to full tail-water pressure at the down-stream end. The hydraulic gradient may be drawn, as in Fig. 64, by considering the pressure head, h_u, at any point, A, on the path of percolation, to be equal to the vertical distance, h_h, from that point to head-water level, less the friction loss, h_f, between the up-stream end of the path of percolation and the point, or,

$$h_u = h_h - h_f.$$

The friction loss, h_f, is proportional to the relative distance through which the water has traveled and the net head, h_n, on the dam, or,

$$h_f = h_n \frac{l_1+l_2+l_3+l_4+l_5+l_7}{l_1+l_2+l_3+l_4+l_5+l_6}.$$

The uplift pressure, p_u, is, for earth foundations, considered as effective over the entire area, therefore,

$$p_u = w_2 h_u = w_2(h_h - h_f),$$

or,

$$p_u = w_2\left(h_h - h_n \frac{l_1+l_2+l_3+l_4+l_5+l_7}{l_1+l_2+l_3+l_4+l_5+l_6}\right). \quad . \quad . \quad . \quad (45)$$

The uplift pressure, p_u, under the apron must be balanced by the weight of the apron and the weight of water on the apron. To allow a factor of safety, the thickness of the apron is usually made at least 30 per cent greater than theoretically necessary. It was shown in Art. 14 that tail-water may not always be counted on to assist in balancing uplift, in which case the presence of tail-water should also be neglected in Eq. (45). The uplift pressure will vary with the changing relative elevations of head- and tail-water, and the weight of the apron and dam should be proportioned for the severest condition.

The thickness of the apron may be reduced, if properly anchored to bearing piles.

When the friction of the foundation is not considered sufficient to prevent sliding, piles may be used, as in Fig. 69, although

it is obvious that vertical piles cannot withstand lateral pressure without deflecting to some extent. Therefore some movement of the dam may be expected, depending on the number of piles provided. For this reason, some of the piles are often inclined in the direction of the resultant pressure.

In the Mathis Dike Dam, Fig. 40, the longitudinal ribs, at the toe and the middle of the base, are intended to assist in preventing the dam from sliding.

In some cases increased resistance to sliding has been obtained by thoroughly anchoring the dam to a deep cut-off wall at the heel of the structure.

CHAPTER XI

FLOOD FLOWS

63. General Considerations. The magnitude of the probable maximum flood to be expected at a dam site has a direct bearing on the safety of the structure. This is particularly true if, as is often the case, the masonry dam is supplemented at one or both ends by an earthen embankment. Although, for a masonry dam, a flood somewhat greater than that for which the structure has been designed may, because of the margin of safety, be passed without failure or even damage, an earthen embankment will almost invariably be breached soon after the water has begun to flow over the top.

As the magnitude of a flood is affected by an infinite variety of conditions, the chances of the maximum possible flood from a given catchment area occurring within the life of the structure is infinitely remote. It therefore remains to adopt, as the designing condition, a flood which is *reasonably* certain of not being exceeded; it being remembered that a flood which would be considered " reasonable " for one dam may be unreasonably large or small for another dam having exactly the same size and characteristics of catchment area. The probability of loss of life and property, of interest on the investment, of the use for which the dam is intended, and other factors, must be considered in deciding on the flood to be accommodated. In other words, as it is impracticable to design any, except the most important of dams, for the maximum possible flood, the problem is resolved into a consideration of how much chance it is reasonable to take. It is safe to say that very few dams in America have been designed to accommodate a flood proportionally as great as that which occurred on the Miami water-shed at Dayton, Ohio, in 1913 (see No. 72 of Fig. 71), which was one of the largest floods this country has ever known.

The maximum flood to be accommodated from a given catchment area may be determined approximately by the following

methods, it seldom, if ever, being possible to make a direct discharge measurement on a record flood at the time of its occurrence:

1. By a study of record high-water marks on the stream in question;
2. By comparison with known record floods from other catchment areas of about the same size and characteristics.

64. High-water Marks. Authentic Federal and State Government records of high-water, extending over long periods, may be obtained for many streams. Such records are also often available from mill operators and the officials of municipalities. In the great majority of cases, however, the determination of the elevation of record high-water must be from the observations and traditions of residents, and from physical indications on the banks of the stream.

Observations and traditions of residents should be regarded with caution. Individual reports of untrained observers are subject to great error and, strange to say, are often of doubtful veracity, as the desire to report a high-water a little higher than that reported by a neighbor is often, among certain classes, greater than the love of the truth. Unfortunately, also, reports are sometimes biased by a desire to give an impression of great or small maximum floods, whichever, in the opinion of the observer, will better serve his interests. However, credence may be given when a number of observations closely agree and are referred to definite objects, such as sills of doors and windows or nails driven for reference.

Confirmation may be obtained from the elevation of depositions of brush, logs, or alluvial matter, scars from floating bodies on banks and large trees, and whatever other indications of high-water may be discovered. High-water, in an alluvial valley which has been formed from the sediment deposited from floods, is, of course, always higher than the surface of the valley.

The elevation of record high-water having been fixed, there are three methods by which an estimate of the corresponding discharge can be made:

1. By determining the corresponding head on a dam which existed at the time of such high-water, from which the discharge over that structure can be computed from one of the well-known weir formulæ.*

* See U. S. Geol. Survey Paper No. 200 by R. E. Horton.

2. If a considerable length of straight river having a nearly uniform cross-section and slope is available, a fairly close estimate of the discharge can be made by use of Kutter's formula for the flow in open channels, particularly if accurate current meter measurements of moderate floods have been made in order to determine the coefficient of roughness of the channel.*
3. By the projection of a rating curve to the elevation of record high-water.† This method, however, is only available as a rough indication of the corresponding flood, unless the cross-section of the river is particularly regular and the discharge measurements used in plotting the curve cover a range including floods amounting to at least 30 per cent of the probable maximum.

65. Comparison with Other Rivers. The recorded maximum floods on a number of streams in the United States are indicated in Table XXV and Fig. 71. From a study of such records and the characteristics of the corresponding catchment basin, many equations for the determination of maximum floods for particular cases have been devised. Mr. W. E. Fuller's paper on flood flows and its accompanying discussion furnish the latest expression of opinion on this subject.‡ Fuller's equation for the maximum flood to be expected from any catchment area is:

$$Q_m = NA^{0.8}(1+0.8 \log T)(1+2A^{-0.3}), \quad . \quad . \quad (46)$$

where Q_m is the maximum flood, in cubic feet per second, likely to be exceeded only once in T years;

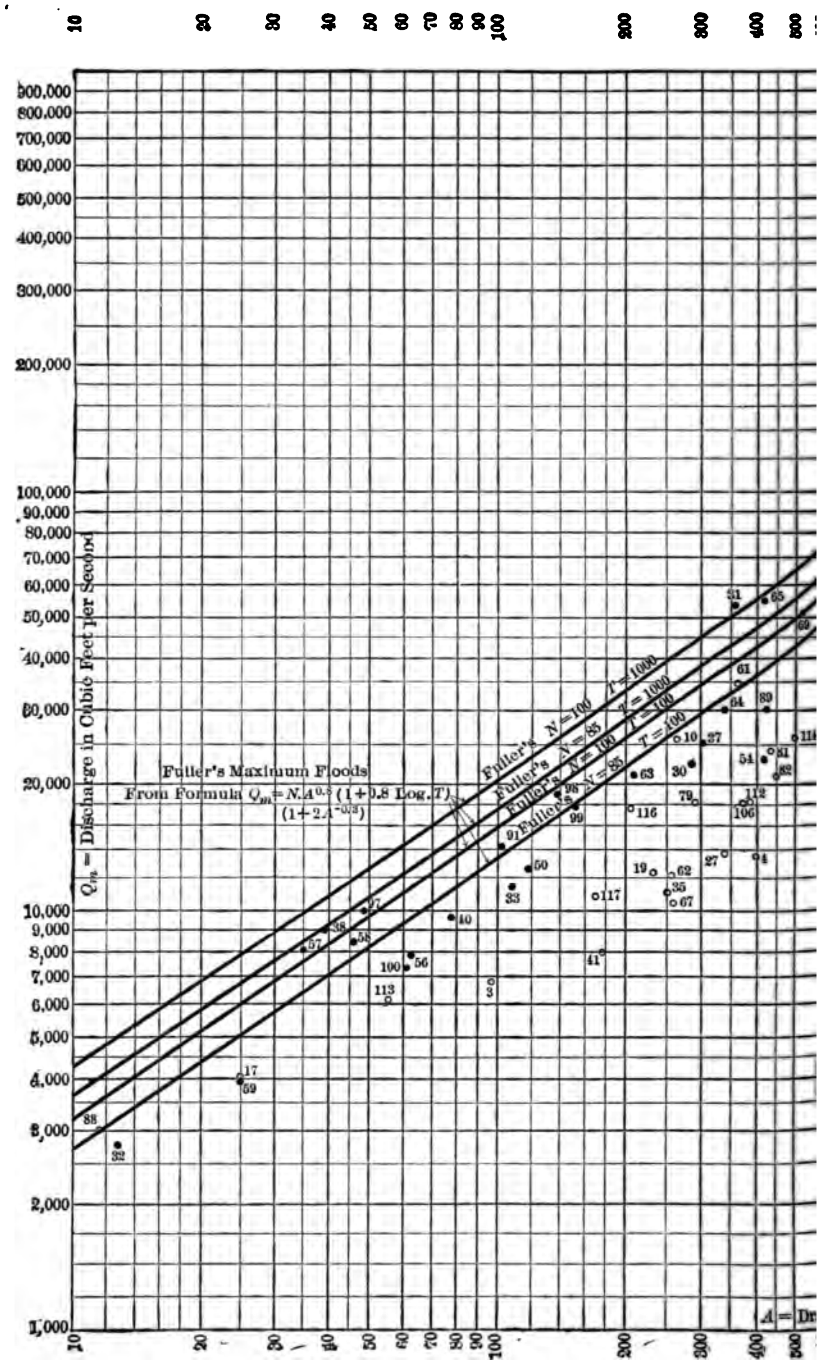
N is a coefficient which is a constant for each water-shed, and depends on its characteristics. This coefficient will be termed the " flood coefficient."

A is the area of the water-shed, in square miles.

* See "River Discharge," by Hoyt and Grover. 3d Edition, 1914. John Wiley & Sons, Inc.

† A rating curve may be defined as a curve the co-ordinates of which represent discharge and corresponding water surface elevations. Such curves are often used in the measurements of daily run-off. The curve is plotted from current-meter measurements of a number of different discharges covering as wide a range as possible.

‡ *Transactions*, Am. Soc. C. E., Vol. LXXVII, p. 564.



Fuller's Maximum Floods
From Formula $Q_m = N.A^{0.8}(1+0.8 \text{ Log. } T)(1+2A^{-0.3})$
Q_m = Discharge in Cubic Feet per Second
Fuller's $N=100$ $T=1000$
Fuller's $N=85$ $T=1000$
Fuller's $N=100$ $T=100$
Fuller's $N=85$ $T=100$
A = Dr

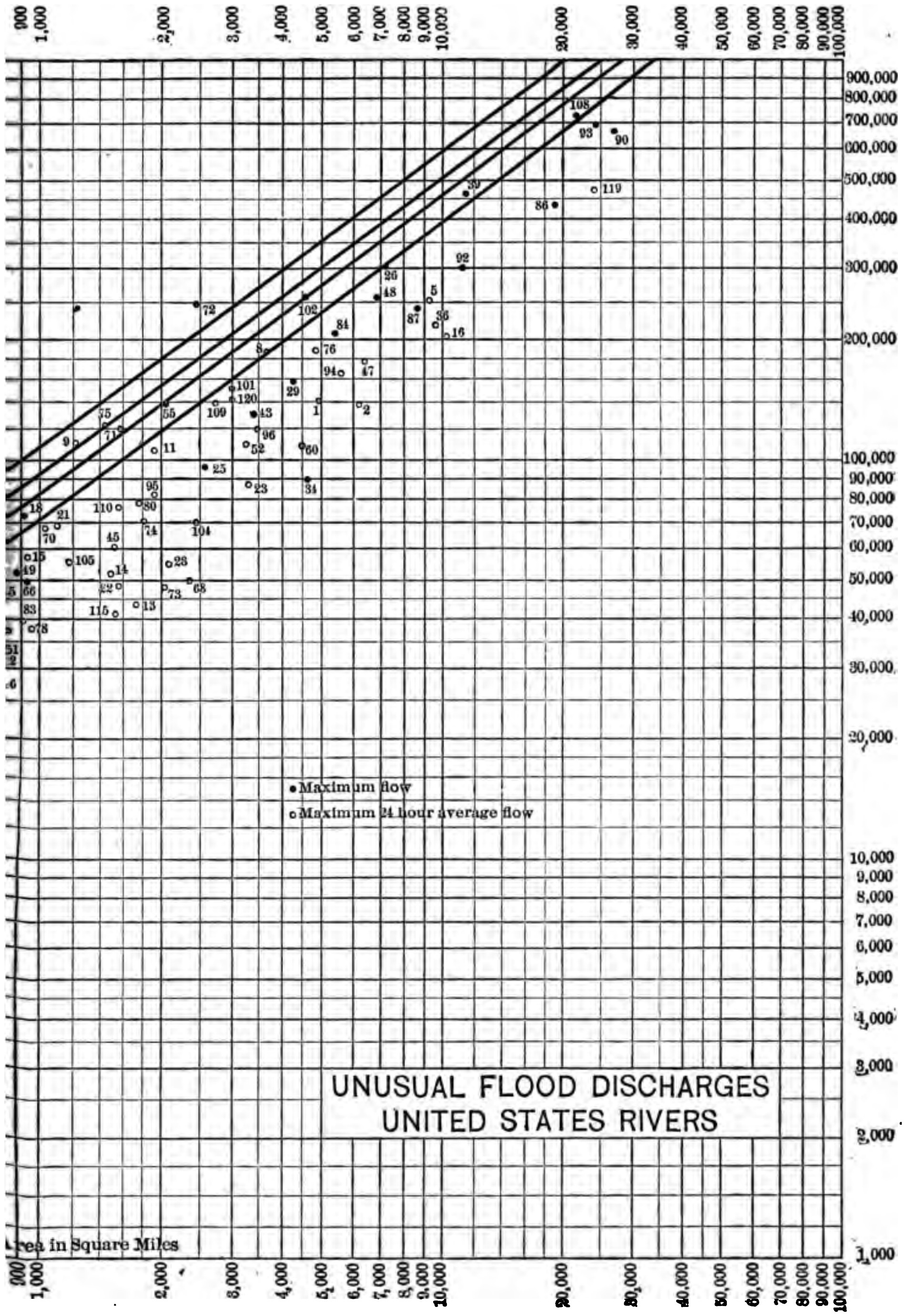


FIG. 71.

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TABLE XXV

UNUSUAL FLOOD DISCHARGES, UNITED STATES RIVERS *

Reference. †	River.	Place	Square Miles of Drainage Area.	Discharge, in Second-feet.
1	Black Warrior	Tuscaloosa, Ala.	4,900	141,000
2	Salt	McDowell, Ariz.	6,260	138,000
3	Malibu Creek	Calabasas, Cal.	97	6,800
4	Mojare	Victorville, Cal.	400	13,400
5	Sacramento	Jellys Ferry, Cal.	9,300	254,000
6	McCloud	Gregory, Cal.	608	41,000
7	Stony Creek	Fruto, Cal.	601	29,300
8	Feather	Oroville, Cal.	3,640	187,000
9	Yuba	Smartville, Cal.	1,220	111,000
10	Bear	Vantrent (above Wheatland), Cal.	263	25,800
11	American	Fairoaks, Cal.	1,910	105,000
12	Putah Creek	Winters, Cal.	805	30,000
13	Kings	Sanger, Cal.	1,740	43,900
14	Tuolumne	La Grange, Cal.	1,500	52,000
15	Stanislaus	Knights Ferry, Cal.	935	57,200
16	Connecticut	Hartford, Conn.	10,200	205,000
17	Requonnock	Conn.	25	4,000 ‡
18	East Branch	Del.	920	72,000 ‡
19	Toccoa	Blue Ridge, Ga.	231	12,300
20	Broad (Ga.)	Carlton, Ga.	762	47,200
21	Oconee	Greensboro, Ga.	1,100	68,200
22	Chattahoochee	Oakdale, Ga.	1,560	48,800
23	Chattahoochee	West Point, Ga.	3,300	88,600
24	Etowah	Canton, Ga.	604	19,300
25	Rhine	Macon, Ga.	2,570	96,500 ‡
26	Savannah	Augusta, Ga.	7,300	310,000 ‡
27	Carrabasset	North Anson, Maine	340	13,700
28	Androscoggin	Rumford Falls, Maine	2,090	55,500
29	Kennebec	Waterville, Maine	4,270	157,000 ‡
30	Piscataquis	Foxcroft, Maine	286	22,200 ‡
31	Westfield	Mass.	356	53,000 ‡
32	Fomer	Above Reservoir, Holyoke Mass.	13	2,840 ‡
33	Nashua	Mass.	109	11,400 ‡
34	Merrimac	Lawrence, Mass	4,640	90,000 ‡
35	Patapsco	Woodstock, Md.	251	11,000
36	Potomac	Point of Rocks, Md.	9,650	219,000
37	Gunpowder	Md.	302	25,000 ‡
38	Lake Roland	Near Baltimore, Md.	39	9,000 ‡
39	Potomac	Great Falls, Md.	11,500	470,000 ‡
40	Rock Creek	D. C.	78	9,800 ‡
41	St. Mary	Near Babb, Mont.	177	7,980
42	Tuckaseegee	Bryson, N. C.	662	38,700 ‡

* Compiled principally from Fuller's "Flood Flows," Transactions, Am.Soc.C.E., Vol. LXXVII.

† See Fig. 71.

‡ Maximum flows. Other discharges are twenty-four hour average flows

TABLE XXV—*Continued*

Reference. †	River.	Place.	Square Miles of Drainage Area.	Discharge, in Second-feet.
43	Gadkin	Salisbury, N. C.	3,400	130,000 ‡
44	Catawba	Morganton, N. C.	758	32,200
45	Catawba	Catawba, N. C.	1,530	61,000
46	Raritan	Bound Brook, N. J.	800	28,500
47	Delaware	Riegelsville, N. J.	6,430	177,000
48	Delaware	Stockton, N. J.	6,850	255,000 ‡
49	Raritan	Bound Brook, N. J.	879	52,000 ‡
50	Ramapo	N. J.	118	12,500 ‡
51	Passaic	Dundee Dam, N. J.	820	35,000 ‡
52	Delaware	Port Jervis, N. Y.	3,250	108,000
53	Delaware, W. B.	Hancock, N. Y.	680	33,700
54	Buffalo Cr.	N. Y.	420	23,000 ‡
55	Chemung	Elmira, N. Y.	2,050	138,000 ‡
56	Nine Mile Cr.	Stittville, N. Y.	63	7,820 ‡
57	Sawkill	45 m. above mouth, N. Y.	35	8,000 ‡
58	Six Mile Cr.	Ithaca, N. Y.	46	8,500 ‡
59	Traut Br.	Brooksport, N. Y.	25	3,950 ‡
60	Hudson	Mechanicsville, N. Y.	4,500	108,000
61	West Canada Cr.	Twin Rock Br., N. Y.	364	34,300
62	East Canada Cr.	Dolgeville, N. Y.	256	12,100
63	Catskill	Woodstock, N. Y.	210	21,000 ‡
64	Croton	Croton Dam, N. Y.	339	30,000 ‡
65	Esopus	Saugerties, N. Y.	417	55,000 ‡
66	Schoharie	N. Y.	930	49,600 ‡
67	Salmon	Pulaski, N. Y.	260	10,500
68	Genesee	Rochester, N. Y.	2,360	50,000
69	Olentangy	Columbus, Ohio	520	51,000
70	Upper Scioto	Waterworks Dam, Ohio	1,030	68,000
71	Lower Scioto	Columbus, Ohio	1,570	119,000
72	Miami	Dayton, Ohio	2,450	246,000 ‡
73	Rogue	Tolo, Ore.	2,020	48,300
74	Umpqua, S. F.	Brockway, Ore.	1,800	70,700
75	Willamette M. F.	Jasper, Ore.	1,450	122,000
76	Willamette	Albany, Ore.	4,860	188,000
77	Willamette, Coast Fork	Goshen, Ore.	690	31,300
78	McKenzie,	Springfield, Ore.	960	37,900
79	Yamhill	Sheridan, Ore.	290	18,100
80	Kiskiminetis	Avonmore, Pa.	1,750	77,700
81	Youghiogheny	Confluence, Pa.	435	24,000
82	Casselman	Confluence, Pa.	450	20,800
83	Clarion	Clarion, Pa.	910	39,300
84	Monongahela	Pa.	5,430	207,000 ‡
85	Youghiogheny	Pa.	782	46,000 ‡
86	Ohio	Pa.	19,000	440,000 ‡
87	Allegheny	Kittanning, Pa.	8,700	241,000 ‡
88	Spring Cr.	Pa.	11.6	3,000 ‡
89	Stony Cr.	Johnstown, Pa.	428	30,000 ‡
90	Susquehanna	McCalls Ferry, Pa.	26,800	671,000 ‡

† See Fig. 71.
‡ Maximum flows. Other discharges are twenty-four hour average flows.

TABLE XXV—*Continued*

Reference.†	River.	Place.	Square Miles of Drainage Area.	Discharge, in Second-feet.
91	Tohickon, Cr.	Point Pleasant, Pa.	102	14,100 ‡
92	Susquehanna	Danville, Pa.	11,100	305,000 ‡
93	Susquehanna	Harrisburg, Pa.	24,000	700,000 ‡
94	Susquehanna W. B.	Williamsport, Pa.	5,640	164,000
95	Schuylkill	Philadelphia, Pa.	1,920	82,200
96	Junia	Newport, Pa.	3,480	118,000
97	Br. Conemaugh	Johnstown, Pa.	49	10,000 ‡
98	Neshaminy Cr.	Below forks, Pa.	139	19,000 ‡
99	Perkiomen	Frederick, Pa.	152	17,600 ‡
100	Flat	R. I.	61	7,350 ‡
101	Catawba	Rockhill, C. S.	2,990	151,000
102	Broad (Car.)	Parr, S. C.	4,570	252,000 ‡
103	Tugalloo	Madison, S. C.	593	21,900
104	Little Tennessee	McGhee, Tenn.	2,470	70,000
105	Hiwassee	Reliance, Tenn.	1,180	55,200
106	Ocoee	McCays, Tenn.	374	18,000
107	Little Tennessee	Judson, Tenn.	675	57,500 ‡
108	Tennessee	Chattanooga, Tenn.	21,400	735,000 ‡
109	New	Radford, Va.	2,720	138,000
110	Shenandoah S. F.	Front Royal, Va.	1,570	76,800
111	James N. F.	Glasgow, Va.	831	37,200
112	Roanoke	Roanoke, Va.	388	18,100
113	Yakima	Martin, Wash.	56	6,150
114	Yakima	Cle Elum, Wash.	500	25,600
115	Yakima	Umtanum, Wash.	1,540	41,000
116	Clealum Lake	Roslyn, Wash.	205	17,700
117	Cedar	Ravenusdale, Wash.	170	10,800
118	Black	Neillsville, Wis.	675	23,100
119	Ohio	Wheeling, W. Va.	23,800	480,000
120	Shenandoah	Millville, W. Va.	3,000	140,000

† See Fig. 71.
‡ Maximum flows. Other discharges are twenty-four hour average flows.

This equation furnishes a means for comparing the maximum floods to be expected from water-sheds of different areas but having the same characteristics.

Aside from the difference in area of the water-shed, two streams may have materially different flood tendencies accounted for by a difference in the characteristics of the water-shed. The flood coefficient, N, in Eq. (46), which is necessary on account of such differences in characteristics, depends on three general conditions, there being several subdivisions, as explained later.

1. The prevailing conditions of rainfall;
2. The storage capacity of the water-shed, or its ability to retain temporarily and distribute the maximum rainfall.

3. The capacity of the water-shed suddenly to release stored waters.

The annual rate of rainfall on any water-shed is not always an indication of the maximum rate or intensity of precipitation which may be expected. A detailed study of the records of the U. S. Weather Bureau for rain-gauging stations on the catchment area and vicinity will prove of material assistance in determining the maximum rate and duration of precipitation to be expected.

Storage, of whatever nature, has a tendency to reduce the size of floods. The storage capacity of the water-shed may be divided into the following items:

1. Storage in reservoirs, lakes, and swampy places;
2. Storage below the ground surface;
3. Storage above the ground surface.

It is seldom that storage below the normal flow line of artificial reservoirs is effective in reducing the peak of large floods, because at such times the reservoirs are very sure to be full due to the excessive flow preceding the peak. Storage above the normal flow line of all bodies of water is always available, as such bodies of water must suffer an increase in surface elevation in order to provide sufficient head at the outlet to accommodate the increasing flow. The percentage of area of reservoirs, lakes, and swampy places has considerable influence on the value of the flood coefficient.

The magnitude of moderate floods is always less on rivers draining large sandy areas, in which the storage capacity below the ground is considerable. Unless, however, such sandy areas are large and extend to the higher elevations of the catchment area, little if any reduction in large floods will be effected, because, at such times, the voids below the ground surface are apt to be completely filled as the result of the excessive precipitation preceding the peak of the rainfall. Storage below the ground, therefore, is usually considered only as increasing the interval between floods.

Storage above the ground is affected by the nature of the vegetation, the shape and slope of the catchment area, and the characteristics of the river bed and banks. It is evident that those characteristics which will permit of rapid run-off of the precipitation to the site of the dam will result in large floods. Rocky

slopes, devoid of vegetation are conducive to quick discharge. Conversely, areas covered with dense vegetation will prove effective in holding back the water and smoothing out the peak of the flood. Heavy underbrush is particularly effective in this respect, as the rivulets are held back by friction in passing around and among the stalks of the plants and such branches as have been beaten down to the ground surface. Practically no water, at the peak of the precipitation, is held back by adherence to leaves and branches above the ground surface. For this reason it is the opinion of many engineers that it is the removal of the dense underbrush rather than the large trees which has increased flood tendencies in districts which have been deforested.

Steep slopes, of course, will produce rapid run-off. Therefore floods from mountainous districts are relatively severe.

In rivers having tributaries extending fan-shaped from a given point, and of approximately the same size, the peak of the flood from each of the tributaries is apt to reach the main stream and the dam at approximately the same time, resulting in relatively large floods. Conversely, when the catchment area is relatively narrow, with tributaties of different sizes discharging into the main stream at regular intervals, the peak of the run-off from the tributary areas will reach the dam at different times, resulting in relatively small floods. A large number of tributaries is also productive of rapid run-off.

Rivers and tributaries which have frequent restricted cross-sectional areas, rough bottoms, and are relatively shallow in comparison with their widths may also be said to have a moderately retarding effect on the rapidity of run-off.

The capacity of the catchment area to release stored water suddenly may be indicated by:

1. The frequency and magnitude of ice and log jams, with consequent danger of release of the impounded waters at or near the peak of the flood.
2. The presence of other dams of questionable strength impounding large volumes of water. A number of well-designed dams have failed on account of the destruction of defective dams above, with a resultant enormous increase in the run-off due to the sudden release of impounded waters.

3. Temporary partial blocking of the flow of the stream, due to lodgment of débris against submerged bridges, and subsequent failure of the bridges, with a release of the impounded waters at or near the peak of the flood.
4. Storage in the form of snow which may be suddenly released by a record precipitation accompanied by a rise in temperature.

Little or nothing has been accomplished which would indicate a definite relation between flood coefficients and characteristics of catchment areas. Until more information has been obtained the matter must be left to the judgment of the engineer. It is probable that, in general, the maximum rate and duration of rainfall, the steepness of the slopes, the shape of the catchment area, and the arrangement of tributaries will, in the order given, have the most influence on the flood tendencies of the stream. The items mentioned as affecting the capacity of the stream to release stored waters suddenly cannot be included in a general classification, as their effect on floods is too uncertain.

An indication of the relative flood tendencies of two streams may sometimes be gleaned from a study of the records of maximum yearly floods when available. Mr. Fuller, in his paper on flood flows, has given a unique approximate method of determining directly the value of the flood coefficient, N, from a study of the maximum yearly floods on the stream in question.

CHAPTER XII

DETAILS AND ACCESSORIES

66. Masonry for Dams. During the latter part of the last century, rubble masonry was used extensively for the construction of dams; but in recent years this type has been practically superseded by concrete and cyclopean concrete masonry.

Cyclopean concrete masonry consists of plain concrete containing a large percentage of irregular stones, or "plums." These stones should be as large as can be economically quarried, transported, and handled; and they should comprise as large a percentage of the mass as possible, consistent with good work. Spalls are rammed into the concrete between the plums. Plums and spalls, of course, are intended to effect a saving in concrete, and, if the work is properly done, should not reduce appreciably the strength of the masonry.

For a small structure, the greatest dimension of any plum should not exceed 20 per cent of the thickness of the structure, provided the masonry is to be stressed to a point approaching a reasonable working limit. A wet concrete is essential to the proper setting of the plums. Fig. 72 shows the cyclopean concrete masonry of the Olive Bridge Dam in the process of construction.

As much as 30 per cent of plums can be used advantageously, but a larger precentage is apt to result in ineffective packing of the concrete. The usual percentage is from 18 to 22.

It has been claimed that a 1 : $2\frac{1}{2}$: 5 or even a 1 : $2\frac{1}{2}$: 4 mix for cyclopean concrete, on account of its greater fluidity, will not only give decidedly better masonry than a 1 : 3 : 6 mix, but will permit of the addition of many more plums and spalls, thus causing an actual decrease in the quantity of cement per cubic yard of masonry. It is also claimed that an aggregate limited to a maximum size of 2 in. will, for the same reason, give better results than the usual limit of 3 in. or greater. The results to be attained, however, will depend decidedly on local conditions and the personal equation of the superintendent, it often being the case that,

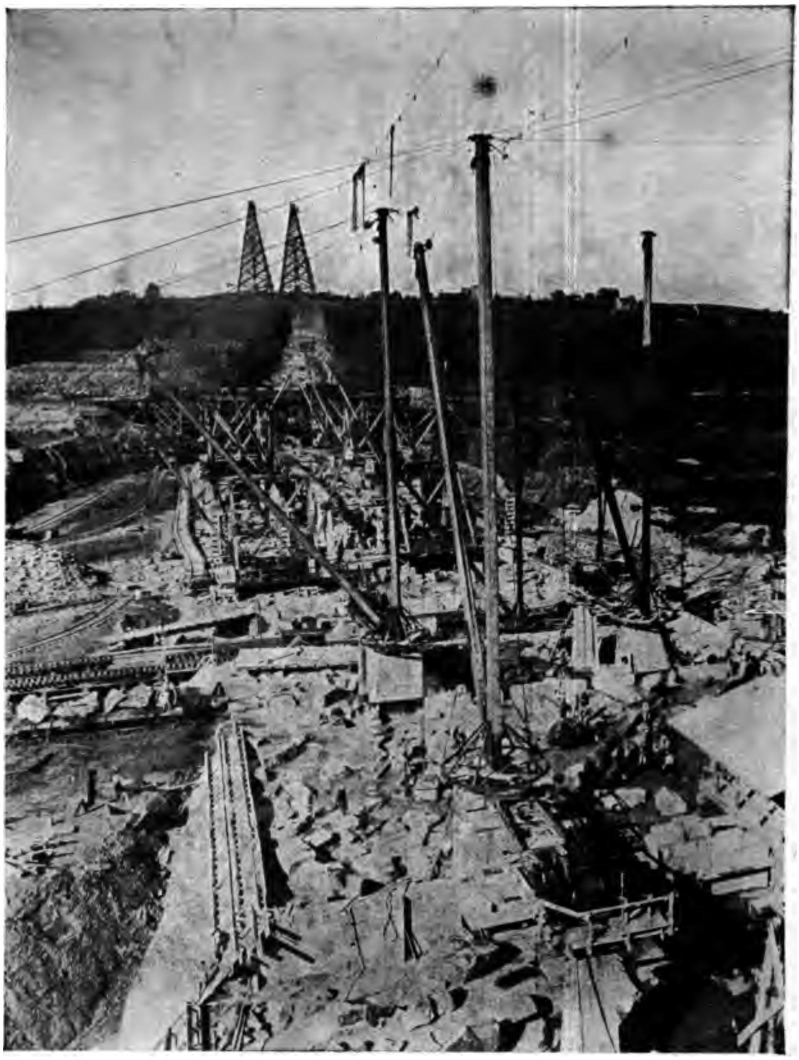


FIG. 72.—The Olive Bridge Dam During Construction. Showing Plums Projecting above Horizontal Joints.

through the lack of material, or by carelessness, the maximum quantities of plums and spalls are not used, and then the richer concrete is of no economical value.

The following proportions of concrete have been commonly used for the various types of concrete dams:

Solid gravity dams and large arch dams,	1 : 3 : 6 cyclopean concrete;
Thin arch dams,	1 : $2\frac{1}{4}$: $4\frac{1}{2}$ to 1 : $2\frac{1}{2}$: 5, plain or reinforced concrete;
Slab decks of hollow dams,	1 : 2 : 4 reinforced concrete;
Arched decks of hollow dams,	1 : 2 : 4 plain or reinforced concrete;
Buttresses of hollow dams,	1 : $2\frac{1}{2}$: 5 to 1 : 3 : 6 reinforced concrete;
Struts of hollow dams,	1 : 2 : 4 reinforced concrete.

To the customary requirements for first-class concrete masonry should be added, for masonry dams, the special provision for the maximum possible bond at horizontal building joints. The importance of this feature, as an additional element of safety, has already been pointed out. The proper treatment in this respect also serves to reduce greatly the leakage through such joints, with a resultant decrease in uplift pressure and prevention of unsightly discolorations of the down-stream face.

Wherever new concrete is to be laid on old, the surface of the latter should be thoroughly cleaned by using stiff brushes and streams of water; the dead cement and laitance should be scraped from the old surface; and the latter should be thoroughly wet just before the new concrete is placed. It is good practice to spread a $\frac{1}{2}$-in. layer of mortar on the old surface immediately before pouring the new concrete.

Care should be taken to have a considerable number of plums project above the surface of horizontal building joints, in order to increase the frictional resistance to sliding. In the absence of plums, it is frequently the practice to groove the building joints, as indicated in Fig. 41.

67. Water-proofing. Except as stated in Art. 69, no special provision need be made for seepage through the masonry of solid gravity and thick arch dams, constructed of 1 : 3 : 6 or richer concrete, other than cutting and working the concrete well against

the forms in order to insure a skin of cement at the face. Such dams will leak to a limited extent, but are tight enough for usual purposes. Leakage usually ceases after a few years, due to the filling of cracks by silt, or by effervescence of magnesia or lime from the cement.

For thin arch dams, where a mixture not richer than 1 : $2\frac{1}{4}$: $4\frac{1}{2}$ is used, it may be necessary to provide an impervious coating on the up-stream face.*

The 1 : 2 : 4 concrete for the decks of hollow dams should be practically water-tight. This, however, necessitates a careful proportioning of the aggregate, a wet mixture, complete incorporation of the materials, careful placing, and thorough puddling and spading. Tight forms are absolutely necessary. For high heads, the horizontal building joints of the decks have sometimes been coated with a layer of tar, asphalt or similar material, from $\frac{1}{8}$ to $\frac{1}{4}$ in. thick.

In general, as much depends on the quality of the labor and superintendence as on the methods specified and the materials used. The author knows of cases where, with skillful workmanship, thin walls of 1 : $2\frac{1}{4}$: $4\frac{1}{2}$, and even 1 : $2\frac{1}{2}$: 5 concrete have been made practically tight without the addition of any water-proofing material or any special treatment of the finished surfaces.

68. Contraction Joints.† All monolithic masonry structures of considerable length will crack because of restrained shrinkage when reductions in temperature occur. Such reductions may be due to atmospheric conditions or to the cooling of the cement which, when setting, attains a high temperature.

Cracks are objectionable, not only on account of their unsightly appearance, but because of their tendency to follow planes of least resistance lying at almost any inclination to the lines of stress.

It is common practice, therefore, to construct the dam in comparatively short sections, thus providing vertical contraction joints in order to localize such cracks and confine them to planes normal to the axis of the dam.

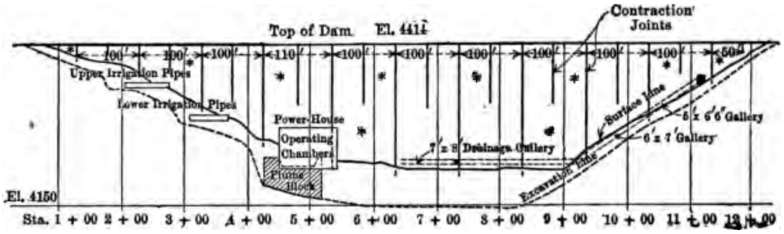
* For descriptions of water-proofing methods see: "Report of Committee on Masonry," Bulletin of the American Railway Engineering Association, Vol. XV, No. 163; "Waterworks Handbook," by Flinn, Weston and Bogert, McGraw-Hill Book Co.; "Concrete, Plain and Reinforced," by Taylor and Thompson, John Wiley & Sons, Inc.

† Sometimes erroneously called "expansion joints."

It has been found that temperature cracks will occur in small walls about every 50 or 60 ft. Experiments show that the internal temperature changes in any masonry structure will vary inversely as its magnitude,* and, for this reason, contraction joints are usually placed farther apart in large dams than in small ones.

For solid dams, a spacing about equal to the mean thickness of the structure, but not less than 40 or 50 ft., seems to have become standard practice. For this there is no logical reason, as the height of the structure is probably also a governing condition. In every case, however, it has apparently proved satisfactory.

Two series of contraction joints have sometimes been used in very high dams, one series extending from the top to about mid-height and the other completely to the foundation. Such an



ELEVATION OF ELEPHANT BUTTE DAM SHOWING CONTRACTION JOINTS.

FIG. 73.

arrangement is indicated in Fig. 73. In order to minimize contraction, the sections marked by asterisks were specified to be built in the coldest season.

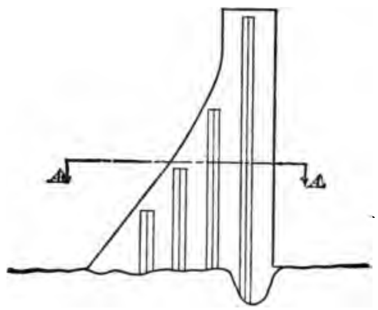
The usual type of contraction joint is indicated in Figs. 74 and 37.

The size and shape of the vertical grooves, or "keyways," vary with the size of the structure, and are essentially a construction feature. It is sufficient only to provide as large a number of corners as is practicable, in order to restrict the flow of water and entangle as much as possible of the sediment and other matter which may be available to tighten the joint gradually.

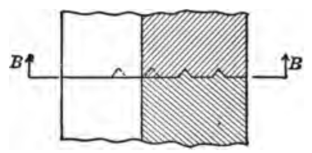
The contraction joint indicated in Fig. 75 is unusually elaborate, and was designed to prevent, to as great an extent as possible,

* "Temperature Changes in Mass Concrete," by Paul and Mayhew, *Transactions*, Am. Soc. C.E., Vol. LXXIX, p. 1225.

all leakage through the joint. The use of concrete blocks for one face of the joint is not usual practice. Rectangular keyways, as here shown, are designed with the idea of providing less area of waterway, on the up- and down-stream sides of the keys, should the joint open a considerable distance. It is thought, however, that the gripping of the keys by the shrinkage of the subsequently poured concrete on the other side of the joint, may destroy its effectiveness and cause cracks through the base of the



SECTION *B-B*



SECTION *A-A*

FIG. 74.—The Usual Type of Contraction Joint.

keys, if not elsewhere in the dam. Of the copper water-stop and the inspection or drainage well, more will be said later.

In flat-deck hollow dams, a contraction joint is usually provided at each buttress. In multiple-arch dams, the arches are usually reinforced continuously through the structure, contraction being taken care of by arch deflection.

Contraction joints should be coated with suitable material in order to prevent adhesion of the masonry and to assist in checking leakage. The author has used a thick coating of coal-tar pitch with complete success. The contraction joints of the Mathis

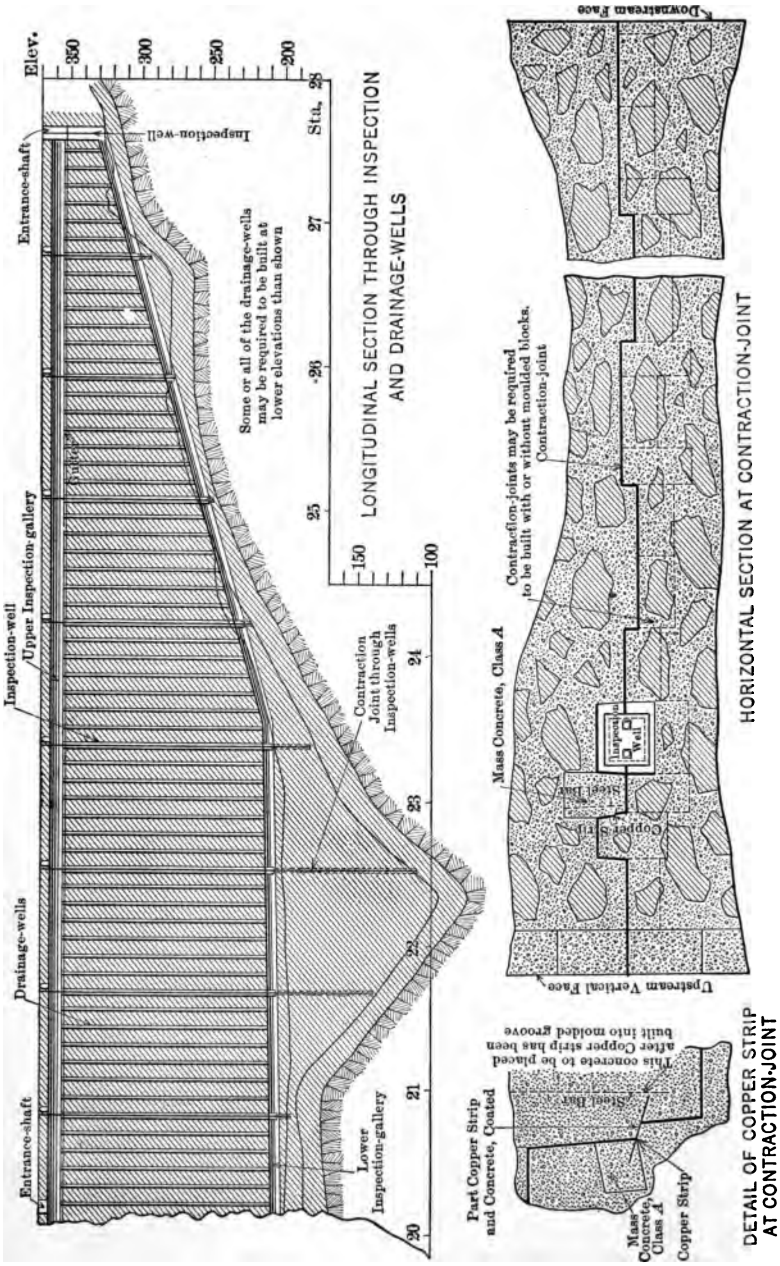


FIG. 75.—Kensico Dam Drainage Wells and Contraction Joints.

Dyke Dam, Fig. 40, were painted with asphaltum. To insure water-tightness, a 6-in. strip of hair felt, laid in hot asphaltum, was placed between the deck and the haunches. In the contraction joints of the Farnham Dam, one face was covered with paper between two coats of pitch, before the concrete on the other side was poured.

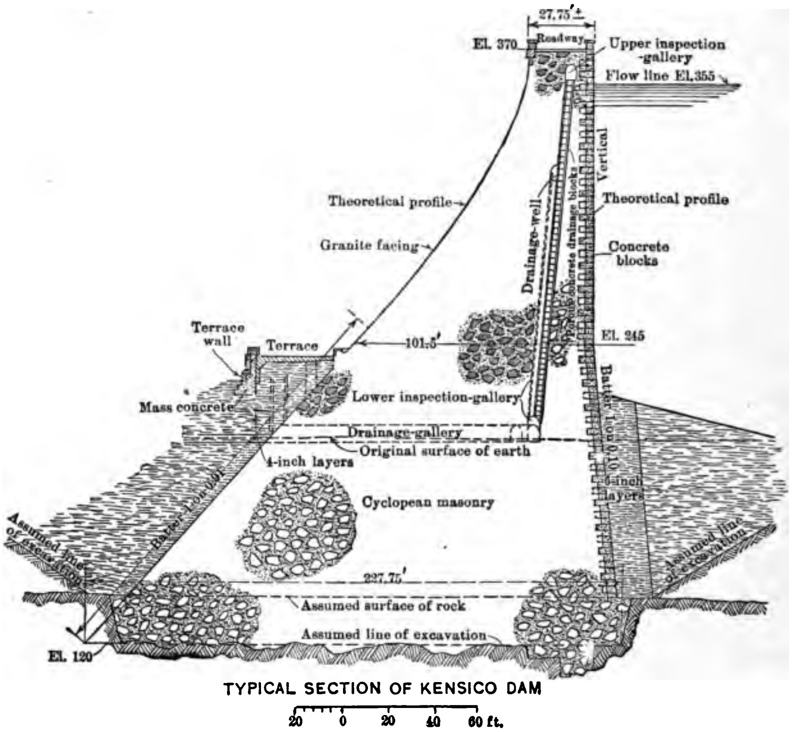


FIG. 76.

69. Drainage Systems. In solid gravity and large arch dams, the usual requirements for first-class masonry and a treatment of horizontal and contraction joints such as previously described will result in a practically impermeable dam. Leakage through contraction joints of the type indicated in Fig. 74 is never excessive, particularly after a few years have elapsed, and may be practically eliminated at once by using metal water-stops, such as shown in Fig. 75. The use of these stops has been confined prin-

cipally to dams in which a discoloration of the down-stream face, due to leakage, would be objectionable, as affecting the appearance of the structure.

Also, for the same reason, seepage through the masonry and horizontal joints has been prevented by special water-tight construction at the up-stream face, backed by a series of vertical drains to intercept and carry away all water which enters the dam. The drains may also be arranged to receive any leakage which finds its way past the metal water-stops.

Except in very high dams, drainage systems in the body of the dam have seldom been used solely for the purpose of preventing uplift on horizontal planes. Unless the head is excessive, properly treated horizontal building joints are sure to be capable of resisting, in adhesion, the pressure of what little water finds its way into the masonry. The conditions within the body of the dam are ideal in comparison with those of the usual foundations, where drainage systems are often necessary. (Art. 61.)

The elaborate system of drainage for the Kensico Dam is indicated in Figs. 75 and 76. The up-stream face of this dam was made relatively less permeable by the use of solid concrete blocks. The drainage and inspection wells were formed by laying up blocks of hollow porous concrete.

70. Architectural Treatment. Except in public works, the architectural appearance of dams is usually neglected. In private enterprises the need for strict economy usually will not warrant the outlay necessary for esthetic treatment; particularly if, as is usual, the project is in a sparsely populated district.

Perhaps the most extensive treatment for architectural effect is that of the Kensico Dam, of the New York City Water Supply System,* (frontispiece). In this case the dam, while serving a useful purpose, was also intended to afford a monumental expression of the magnitude and importance of the largest municipal water-supply system ever constructed. The entire down-stream face of the dam is covered with a layer of granite masonry, arranged in panels and surmounted by a continuous cut-stone cornice. A close view of the facing is shown in Fig. 77. Extensive landscape work, with terraces, ramps, and artificial pools, imparts a pleasing appearance to the structure.

* "Architecture of Kensico Dam," by A. D. Flinn, *Engineering News*, Vol. LXXIV, p. 433.

To facilitate construction, the concrete heart of the dam was completed before the setting of the face work was started. The concrete was built in steps, arranged to receive the stone facing.

The style of ornamentation to be adopted must be in keeping with the dignity of the structure. The most appropriate treatment also depends, to a large extent, on the appearance of the neighboring landscape.

FIG. 77.—Granite Facing of the Kensico Dam.

Considerable opportunity for esthetic treatment was afforded at the site of the City Reservoir, No. 3, at Portland, Oregon, Figs. 78 and 79. The type of architecture is admirably in keeping with the beauty of the surrounding park.

On the other hand, the rugged character of the sites of the Salmon River and Elephant Butte Dams, Figs. 80 and 81, necessitated a more simple, massive, and dignified treatment. That of the Salmon River Dam is thought to be the better of the two.

For a solid gravity dam, the weight of copings, pilasters, etc., at the top, being effective in adding to the stability, should not be

Fig. 78.—Near View of City Reservoir No. 3 Dam, Portland, Oregon.

considered an extra expense, except as to the increased cost of forming and placing the necessary material. In a hollow dam, the weight of water, and not masonry, offers the chief resistance to failure, so that a greater expense is attached to such embellish-

ments. An increased thickness, at the top of an arch dam, in the form of copings or cornices, adds materially to the stiffness of the arch.

A water-tight structure is essential, if a pleasing appearance is desired. The slightest leak will not only result in a discoloration of the down-stream face, on account of the darker color of damp

FIG. 79.—Distant View of City Reservoir No. 3 Dam, Portland, Oregon.

masonry, but will pave the way for a later deposit of laitance, which is even more objectionable.*

71. The Regulation of High-water Surface. The land to be purchased or controlled for a reservoir must include the area which will be covered at the time of maximum flood. Obviously, the water stored between the elevation of the crest of the dam and that of highest water seldom serves a useful purpose, and many devices have been used to keep the maximum rise of water surface as small as possible.

* See Fig. 78.

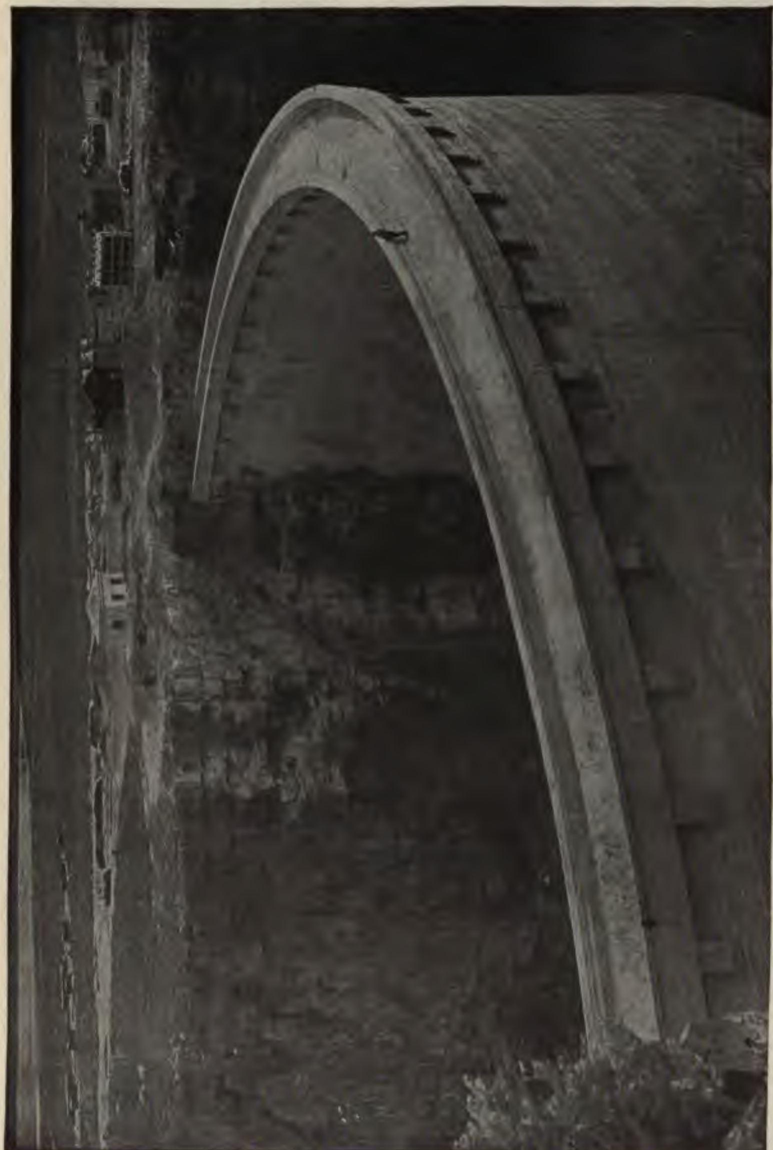


FIG. 80.—Salmon River Dam, Idaho.

FIG. 81.—Elephant Butte Dam, Rio Grande Project, New Mexico.

Flood regulators, in almost unlimited varieties of forms,* have been used. Brief attention will be given here to only a few of the types which have become common in America.

Figs. 82, 82*a* and 83 show typical examples of crest gates. The gates are operated from an elevated platform, the openings being maintained at all times at an area sufficient to provide practically constant water surface elevation in the reservoir. The gates, when entirely raised, should have sufficient capacity to discharge the entire flood, and, if fragile, should be high enough to be clear of large trees and other heavy objects which may be brought down by floods.

The piers may be designed as separate gravity segments down to the rock surface, or as reinforced concrete cantilevers relying for stability on the excessive weight of the adjacent overflow portions.

Vertical grooves are usually provided in the piers, a short distance up-stream from the gates, to facilitate the placing of stop-logs when inspection or repairs of the gates become necessary.

The rectangular type (Fig. 82), varies considerably in detail, according to local conditions and the judgment of the designer, not only in the nature of the materials used, but in the types of bearings, operating mechanism, and the size of openings. Designs and estimates may be obtained from a number of manufacturers who make a specialty of work of this class.

Probably the most common type of crest gate, at the present time, is the sector or " Taintor " gate, Figs. 83, 70 and 68. In this type the pressure of the water, passing through the pivots, causes no resistance to opening or closing, except that of the inconsiderable friction at the pivots and the sealing strips at the sides. Practically, the only force to be controlled by the operating mechanism is the weight of the gate.

Small gates of this type have been made of wood, but for the larger sizes, a steel framework with a steel or wood water face is usually adopted.

Water-tightness at the sides of the Great Falls gates (Fig. 83), was obtained by fastening to them strips of 8-in. five-ply rubber belting which slide on the faces of the concrete piers. Local smoothness of the concrete was obtained by nailing $\frac{1}{4}$-in. smooth steel plates to the inside of the forms. The plates were removed

* See "Improvement of Rivers," by Thomas and Watt, 2d Edition. John Wiley & Sons, Inc., 1913.

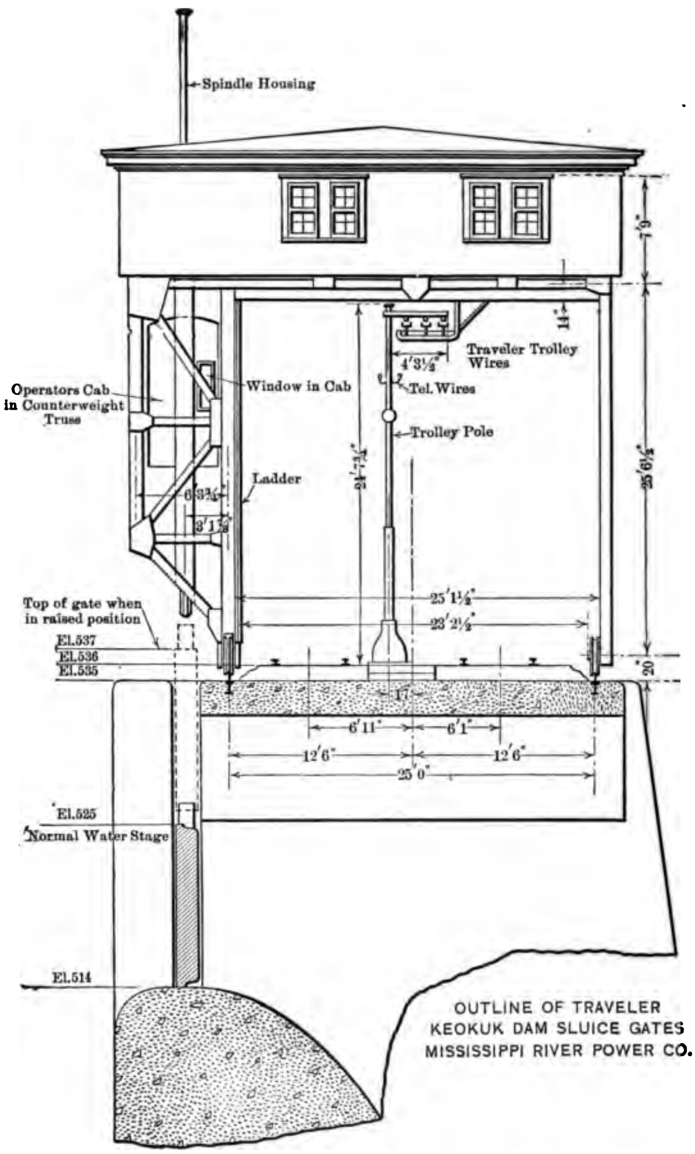

Spindle Housing
7'9"
14"
Traveler Trolley Wires
4'3½"
Tel. Wires
Window in Cab
Operators Cab in Counterweight Truss
Trolley Pole
Ladder
24'7¾"
25'6½"
6'3¾"
3'1½"
Top of gate when in raised position
25'1½"
23'2½"
El.537
El.536
El.535
20"
17"
6'11"
6'1"
12'6"
12'6"
25'0"
El.525
Normal Water Stage
El.514
OUTLINE OF TRAVELER
KEOKUK DAM SLUICE GATES
MISSISSIPPI RIVER POWER CO.

FIG. 82.

Fig. 82a.—Up-stream View of Keokuk Dam Crest Gates, Showing Concrete Counterweight and Steel Gate.

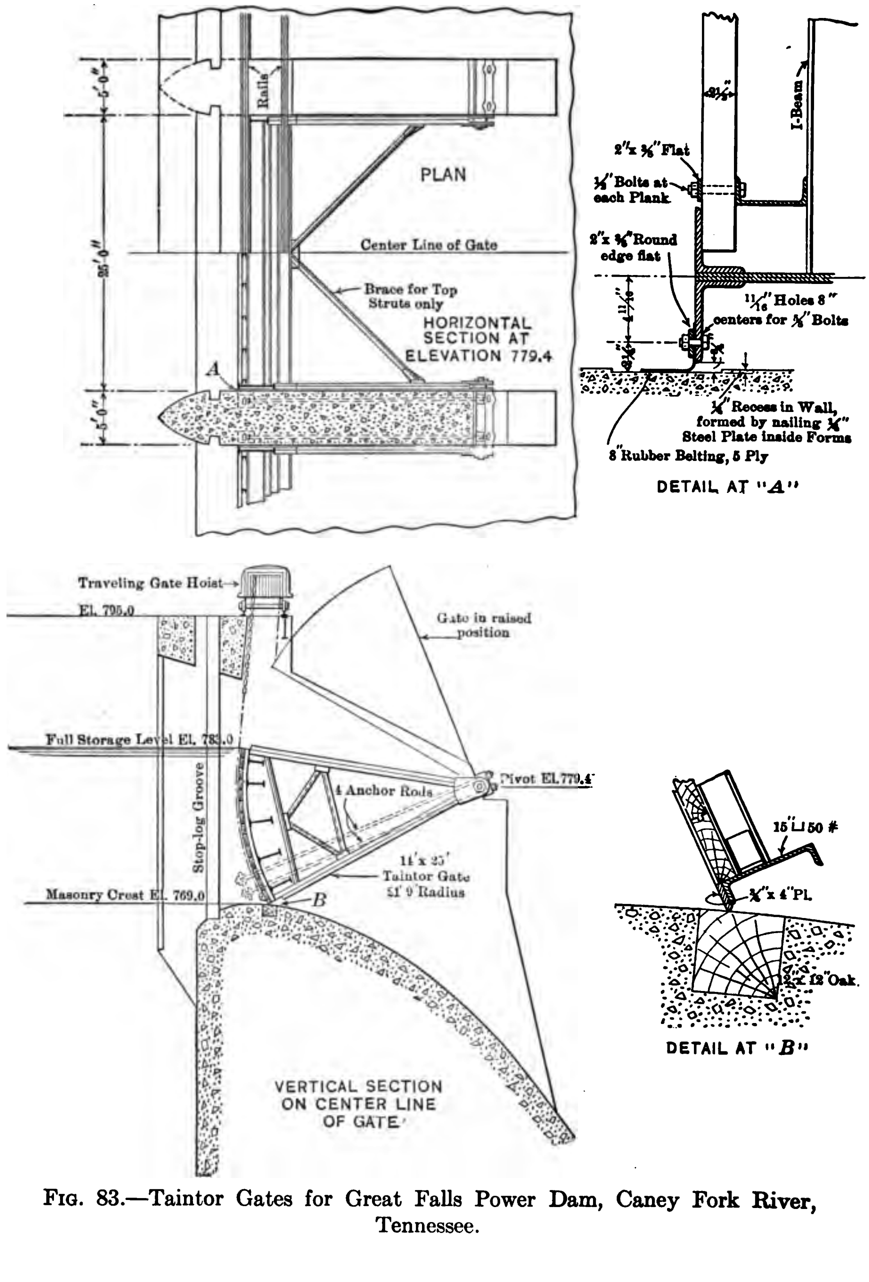


FIG. 83.—Taintor Gates for Great Falls Power Dam, Caney Fork River, Tennessee.

with the forms. For the bottom seal, the lower edges of the gates were planed to a sharp edge. In lowering, this edge cuts through irregularities in the sills, small pieces of wood, and other débris, and the gates come to a continuous bearing.

The pivot pins are usually made to cantilever out from the piers, although they sometimes bear on girders spanning from pier to pier, if danger from floating débris is not feared. The bearing boxes are usually babbited or lined with bronze, in order to prevent them from rusting against the steel pins.

In a plant where a large number of Taintor gates are used, operation is usually effected by one or more traveling hoists to which motors are attached. In cold climates steam pipes should be provided in order to prevent the gates from freezing tight.

Objections have been raised to all form of crest gates which are not entirely automatic in operation, particularly if subjected to ice conditions, on the ground that constant vigilance is necessary for their successful operation. Such antagonism, however, has not prevented the use of non-automatic crest gates in a great many cases, particularly where a sufficient force of men is always available for emergency operation. A few automatic crest gates have been adopted, but their use has not become common.

Fig. 84 shows a typical sluice through the body of the dam. Such sluices may serve a variety of purposes, and are sometimes relied on to pass considerable of the flood flow and thus reduce, in a measure, the rise of water surface during floods. It is seldom found economical to provide a sufficient number of sluices to take the maximum flood flow, as in the case of crest gates.

The Stevens Creek sluice gates are made so as to split, as indicated in Fig. 84, to facilitate removal to the passageway through the water-tight bulkhead. The gates are made accessible by lowering a weighted timber stop gate between the projections on the up-stream face of the dam. If the leakage through the sluice is too great to permit of the stop gate being lowered against the flow, it may be lowered in a horizontal position to an elevation slightly above the top of the sluice entrance and then allowed to swing down over the opening. The sluice is protected from erosion near the gate by a cast-iron lining. Should tail-water rise above the level of the passageway floor, suction at the contracted throat of the sluice will effectually remove all leakage through the drainage pipe provided for that purpose.

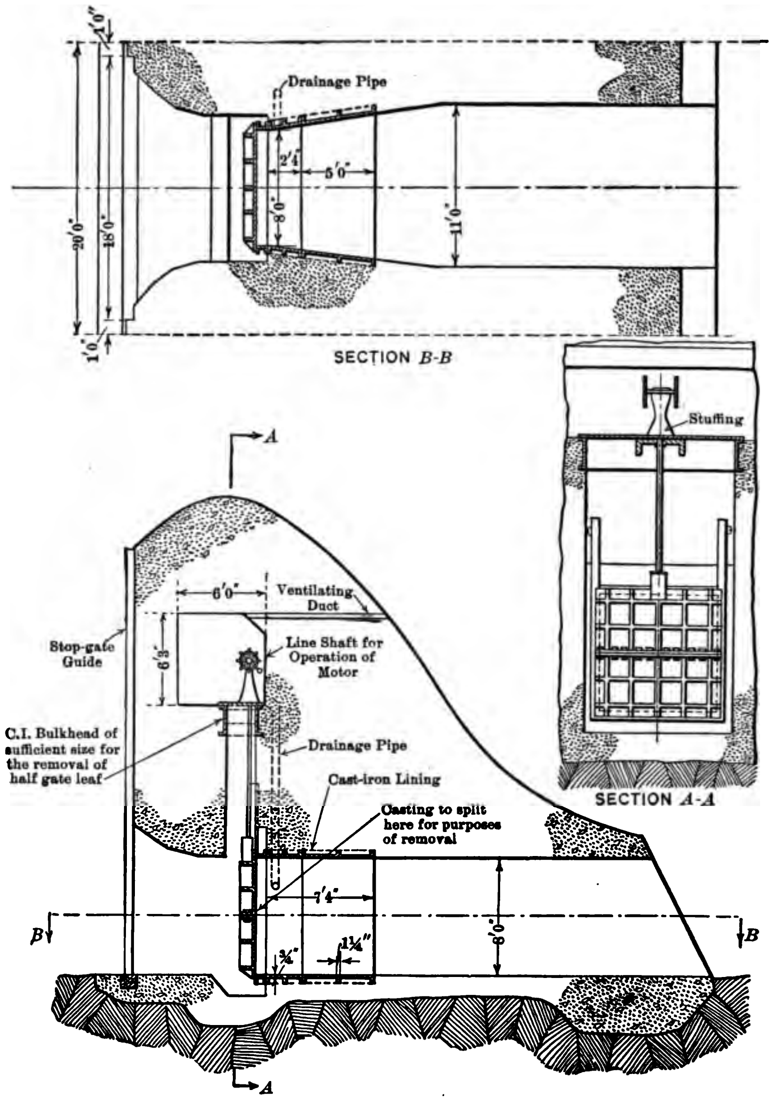


FIG. 84.—Arrangement of 8'×8' Sluice-gates, Stevens Creek Development on Savannah River.

Gates of this type should be made unusually strong in every particular. The disturbance of the water at part gate opening seems to impose a duty on the several parts far greater than the capacity of the manufacturer's usual stock patterns.

Considerable trouble may be caused by logs lodging in the sluice and preventing the gate from closing. Rack bars of 6- to 12-in. spacing are usually placed above the gates. They should be as far as possible from the gates, as cases have been known where logs, although having caught on the racks, have extended through them far enough to rest on the gate sill. For this reason some engineers incline to the opinion that racks are a menace rather than a protection, because, without the racks, the logs would have been carried through the sluice. The use of racks, however, is still quite common. There is a great deal of room for improvement in this feature, as the gates are relatively inaccessible, and, therefore, expensive to maintain and repair.

Fig. 85 shows a typical system of flash-boards with which the normal spillway crest is lowered automatically at times of excess flow. In this country, this is the most common of all devices designed to control the elevation of flood-water surfaces. Although, in certain cases, flash-boards are rather expensive to maintain, owing to frequent renewals, they have the advantage of relatively small first cost and the certainty of automatic operation.

The flash-boards are supported by round steel pins which are inserted loosely into sockets set in the masonry crest of the dam. The boards are usually fastened loosely by wire nails bent around the pins, or by wire loops passing around the pins and fastened to nails driven in the boards. The pins are designed with a reasonable factor of safety, for water at full storage level, but are calculated to bend over and clear the masonry crest, if the boards are not removed, when a severe flood occurs. After the flood passes, the pins are heated, straightened and replaced, and new boards put in place. The boards are usually fastened loosely enough to be removed by the flood, and are seldom recovered.

When increases of flow can be anticipated and sufficient time is available, the boards and pins may be removed by hand before the water surface rises high enough to bend them over. In order to facilitate the handling of a barge for removing and restoring the flash-boards, it may sometimes be found desirable to provide sockets at intervals into which mooring pins may be set. These

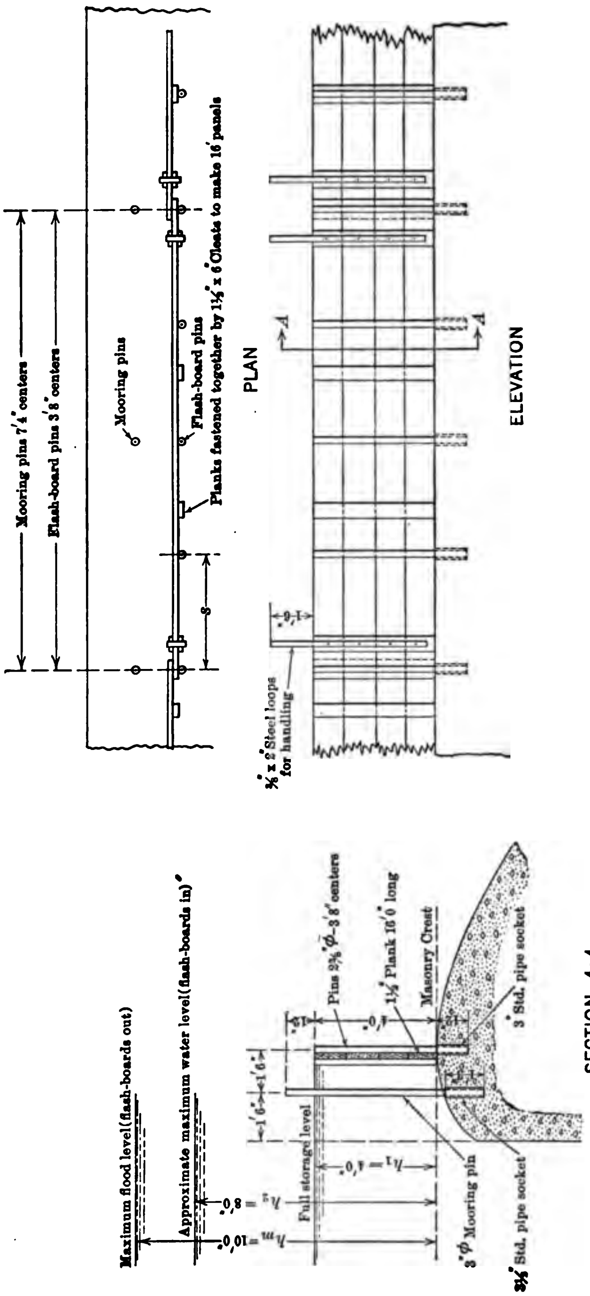


FIG. 85.—A Typical Flash-board Installation.

are indicated in Fig. 85. The boards and pins are sometimes manipulated from an overhead platform or bridge provided for that purpose.

Where it is possible to remove the boards in advance of floods, they are usually built in panels, as indicated, and provided with handles. Handles of the type shown are not permissible if considerable drift is anticipated, as this may collect and cause premature failure.

The boards are sometimes provided with planed edges in order to reduce leakage, but more often are unplaned, and ashes or similar caulking materials are swept along the joints.

As indicated in Fig. 85,

Let d = the diameter of solid pins, in inches;
s = the spacing of solid pins, in feet;
h_1 = the height of solid pins, in feet;
h_2 = depth, in feet, of water above the masonry crest;
h_m = the depth, in feet, of water above the masonry crest corresponding to the maximum flood;
f = the bending stress in the pins, in pounds per square inch;
w_2 = weight of one cubic foot of water = 62.5 lb.

The bending moment in each pin for the water surface at full storage level is

$$M = \frac{w_2 h_1^3 s}{6} = 10.42 h_1^3 s \quad . \; . \; . \quad \text{ft.-lb.}$$

Within the elastic limit, the moment of resistance of the pin, in foot-pounds is,

$$0.00818 f d^3.$$

Equating this to the bending moment, and solving for d, there results,

$$d = \sqrt[3]{\frac{1275 h_1^3 s}{f}}.$$

The use of this equation will provide for the pins having a reasonable stress, say two-thirds of the elastic limit, when the pond is at full storage level.

Before the pins are completely bent over, they are stressed far beyond their elastic limit, and on this account the ordinary theory of flexure is quite inaccurate. Because of this and other uncertainties, it is impossible to write an accurate equation to indicate at what stage of water surface the pins will bend over, and, therefore, we must rely on experimental data. The author has found that, if pins of medium steel are stressed theoretically to about two-thirds of their elastic limit when the water surface is at full storage level, they will ordinarily not withstand a head, h_2, above the masonry crest greater than twice their height, and that they usually bend over for heads equal to from 1.5 to 1.75 of their height, depending on the vacuum which forms under the overflow.* To be amply safe, the height of the flash-boards should not ordinarily be more than about 40 per cent of the head, h_m, corresponding to the maximum flood. This will leave a margin of about $0.2h_1$ between maximum flood level and the highest level at which it is thought the pins will fail.

The pins may then be expected to accommodate, without failure, a flood equal to about one-sixth to one-eleventh of the maximum expected flood, and, if not removed when necessary, will usually bend over several times in the average year unless a part of the moderate floods can be accommodated through sluices in the dam, through turbines, or by other means, or unless the reservoir is exceptionally large.

If a greater degree of accuracy is desired, it becomes necessary to test out samples of the pins under actual operating conditions. Experiments of this character are inexpensive, if a log chute, fishway, or wasteway has been provided in the crest of the dam. In any event, a considerable expenditure will be warranted if a large reduction in land to be purchased can be effected. For greater accuracy, the pins are sometimes grooved to exact diameter at the elevation of the masonry crest.

Pipes have sometimes been used in place of solid pins. If the pipe pins are short in proportion to their diameter, they may fail by buckling at their supports, and go out sooner than solid pins having the same initial stress.

Many types of automatic hinged flash-boards have been used, in which actual failure of the material does not occur when the flash-boards collapse. Their use, however, has not become com-

* See Art. 17.

mon, except on some of the very low river improvement dams built by the Federal Government.

A radically different type of regulator, which seems to be gaining favor, is that of the siphon spillway. Fig. 86 shows the siphon spillway of the Tennessee Power Company on the Ocoee River, Tennessee.*

In a siphon spillway, the water flows through a closed conduit, producing a suction head that largely increases the velocity, and consequently the discharge per unit area.

Under normal conditions, the water, at the Ocoee Siphons, stands at El. 1089.2, or slightly trickles over the lip or crest of the spillway. When the water surface rises, the discharge over the crest increases and the water strikes the far side of the

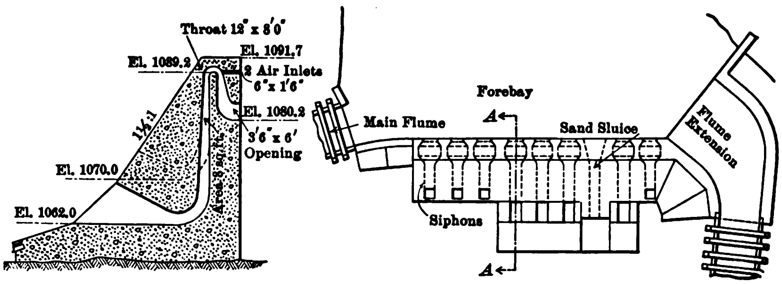


FIG. 86.—Siphon Spillway, Ocoee River Development, Tennessee.

lower siphon legs. When the water surface reaches an elevation of about 1089.55, or slightly above the top of the air inlets, the air thus confined in the top of the siphons, as well as that in the lower legs, is quickly entrained and ejected and the siphons prime. The suction thus produced increases the velocity to that corresponding to an effective head equal to the difference in elevation between the water surface in the pond and the center line of the siphon outlets, less the head expended in friction within the siphons. If the full discharge of the siphons is greater than the total flow, the water surface in the pond will recede. When the upper parts of the air inlets become exposed, the air drawn in by the suction reduces the efficiency of the siphons until the discharge is automatically diminished to that required for stationary

* *Engineering Record*, Vol. LXXII, p. 567.

FIG. 87.—Down-stream View of Ocoee River Siphon Spillway.

FIG. 88.—Up-stream View of Ocoee River Siphon Spillway.

water surface in the pond. If a sufficient quantity of air is drawn through the inlets, however, the siphonic action will be broken. The water surface in the pond will then rise again, and the operation of priming will be repeated.

By virtue of their large capacity, relative to that of a simple weir, of the same crest head, siphons may be used to limit the rise of the water surface in the pond during floods, and are particularly serviceable in cases where the fluctuations in discharge are rapid, as at the end of a long flume supplying hydraulic turbines.

If properly proportioned, they will prime quickly enough for all practical purposes.

The discharge of a siphon of this type may be computed from the ordinary equation for flow through short tubes,

$$Q = CA\sqrt{2gh},$$

where h = the gross head on the siphons, in feet from the water surface to the center line of the discharge;
A = the area at the throat, in square feet;
g = the acceleration of gravity = 32.2;
Q = the discharge, in cubic feet per second; and
C = a coefficient depending on the characteristics of the siphon.

Tests by the author on the Ocoee Siphons indicated a value of C of about 0.65, and this figure has been fairly well substantiated by other tests on similar siphons.

Provision should always be made to by-pass ice cakes and débris which, otherwise, might clog the siphons. The throat should be protected from erosion by a cast-iron lining.

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